

①

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エ. 60 オ. $-\frac{3}{2}$ カ. $\sqrt{3} + \frac{1}{2}$

②

真数条件より $x > 0, y > 0, (3x + y)^2 > 0$

よって $x > 0, y > 0$

$$\begin{aligned}\log_3 xy &= \frac{\log_3 (3x + y)^2}{\log_3 9} \\ &= \frac{2\log_3 (3x + y)}{2} \\ &= \log_3 (3x + y)\end{aligned}$$

$$\therefore xy = 3x + y$$

$$\therefore xy - 3x - y = 0$$

$$\therefore (x-1)(y-3) = 3$$

$x-1, y-3$ は共に整数で,

$$x-1 > -1, y-3 > -3$$

よって, $(x-1, y-3) = (1, 3), (3, 1)$

$$\therefore (x, y) = (2, 6), (4, 4) \dots (\text{答})$$

③

(1) $f'(x) = 3x^2 - 2ax + b$ は

$x=1, x=2$ を解にもつ。

$$f'(x) = 3(x-1)(x-2) = 3x^2 - 9x + 6$$

$$\therefore a = \frac{9}{2}, b = 6$$

$$f(x) = x^3 - \frac{9}{2}x^2 + 6x$$

x	...	1	...	2	...
$f'(x)$	+	0	-	0	+
$f(x)$	$\nearrow$	$\frac{5}{2}$	$\searrow$	2	$\nearrow$

上より極大値 $\frac{5}{2}$, 極小値 2

$$(2) x^3 - f(x) = \frac{9}{2}x^2 - 6x = \frac{3}{2}x(3x - 4)$$

$$1 \leq x \leq \frac{4}{3} \text{ で } x^3 - f(x) \leq 0$$

$$\frac{4}{3} \leq x \text{ で } x^3 - f(x) \geq 0$$

$$\begin{aligned}\text{与式} &= \int_1^{\frac{4}{3}} \left(-\frac{9}{2}x^2 + 6x \right) dx + \int_{\frac{4}{3}}^2 \left(\frac{9}{2}x^2 - 6x \right) dx \\ &= \left[-\frac{3}{2}x^3 + 3x^2 \right]_1^{\frac{4}{3}} + \left[\frac{3}{2}x^3 - 3x^2 \right]_{\frac{4}{3}}^2 \\ &= -2 \times \frac{3}{2} \times \frac{4}{3} \times \frac{4}{3} \times \frac{4}{3} + 2 \times 3 \times \frac{4}{3} \times \frac{4}{3} + \frac{3}{2} \\ &\quad - 3 + 3 \times 4 - 3 \times 4 \\ &= -\frac{64}{9} + \frac{96}{9} - \frac{3}{2} = \frac{32}{9} - \frac{3}{2} \\ &= \frac{64 - 27}{18} = \frac{37}{18} \dots (\text{答})\end{aligned}$$