

$$[I] \quad (1) \quad \text{ア. } 2n+1 \quad \text{イ. } n^2$$

$$\quad \quad \quad \text{ウ. } \frac{n(n-1)(2n-1)}{6}$$

$$(2) \quad \text{I. } 3\sqrt{2} \quad \text{オ. } \frac{37}{4} \quad \text{カ. } -3$$

$$(3) \quad \text{キ. } 10 \quad \text{ク. } 11 \quad \text{ケ. } 12$$

$$\quad \quad \quad \text{コ. } 11$$

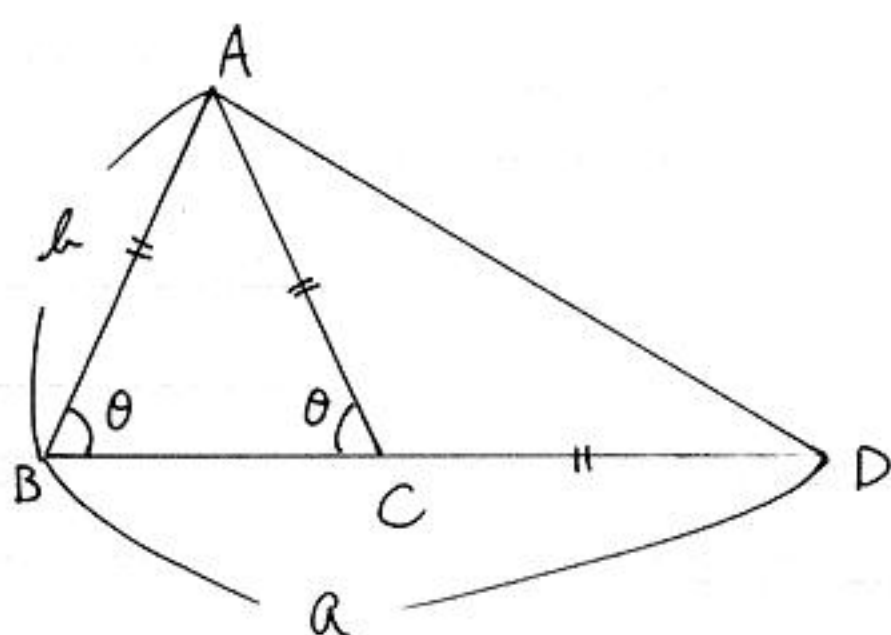
[II] (1) 3通り

(2) 面積 $\frac{3\sqrt{3}}{4}$, 確率 $\frac{1}{10}$

(3) 面積 $\frac{\sqrt{3}}{4}$, 確率 $\frac{3}{10}$

(4) $\frac{9\sqrt{3}}{20}$

[Ⅲ]



(1) $AB = b$ とおくとき、

$$2b \cos \theta + b = a$$

$$\therefore b = \frac{a}{(2 \cos \theta + 1)}$$

よって、

$$\triangle ABD = \frac{1}{2} a \cdot b \cdot \sin \theta$$

$$= \frac{a^2 \sin \theta}{2(2 \cos \theta + 1)} \dots\dots \left(\frac{54}{2}\right)$$

(2) $\triangle ABC : \triangle ACD = \sqrt{3} : 1$ より、

$$BC : CD = \sqrt{3} : 1$$

よって、

$$2b \cos \theta : b = \sqrt{3} : 1$$

$$\Leftrightarrow \cos \theta = \frac{\sqrt{3}}{2}$$

$$\therefore \theta = 30^\circ$$

$$\angle BAC = 180^\circ - 2\theta = 120^\circ$$

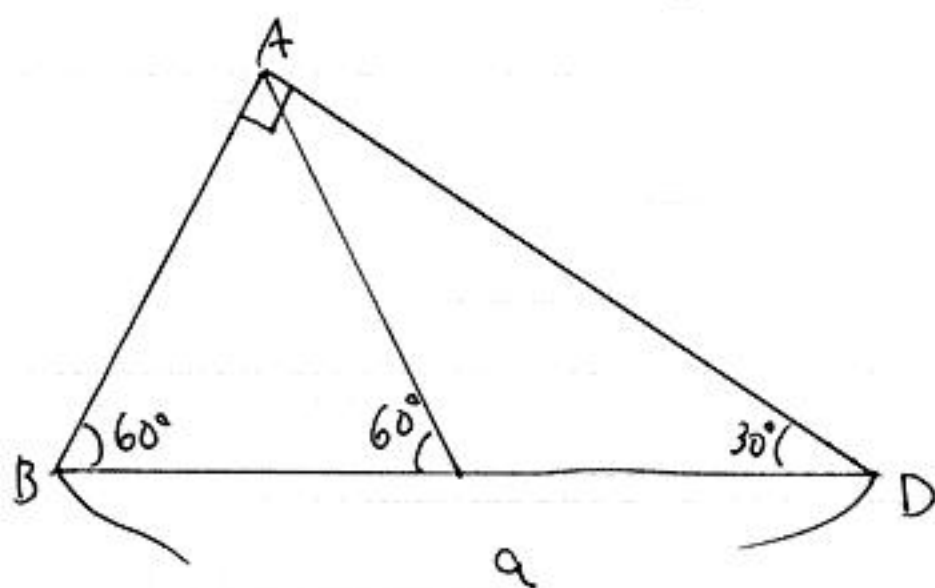
$$\angle CAD = \frac{1}{2} \angle ACB = \frac{1}{2} \theta = 15^\circ$$

$$\therefore \angle BAD = \angle BAC + \angle CAD = 135^\circ \dots\dots \left(\frac{55}{2}\right)$$

$$(3) \triangle ABD = \frac{a^2 \sin \theta}{2(2 \cos \theta + 1)} = \frac{\sqrt{3}}{8} a^2 \text{ より、}$$

$$\frac{\sqrt{3}}{2} \cos \theta + \frac{\sqrt{3}}{4} = \sin \theta \quad \therefore \theta = 60^\circ$$

よって、下図のようになり、



$$AD = a \cos 30^\circ = \frac{\sqrt{3}}{2} a \dots\dots \left(\frac{56}{2}\right)$$