[1]

(1)
$$l:\frac{\sqrt{3}}{2}x-\frac{1}{2}y=1$$
$$\Longleftrightarrow y=\sqrt{3}x-2$$

ここで，

$$ax^2+1=\sqrt{3}x-2$$
$$\Longleftrightarrow ax^2-\sqrt{3}x+3=0 \quad \cdots\cdots①$$

①の判別式を D とすると，

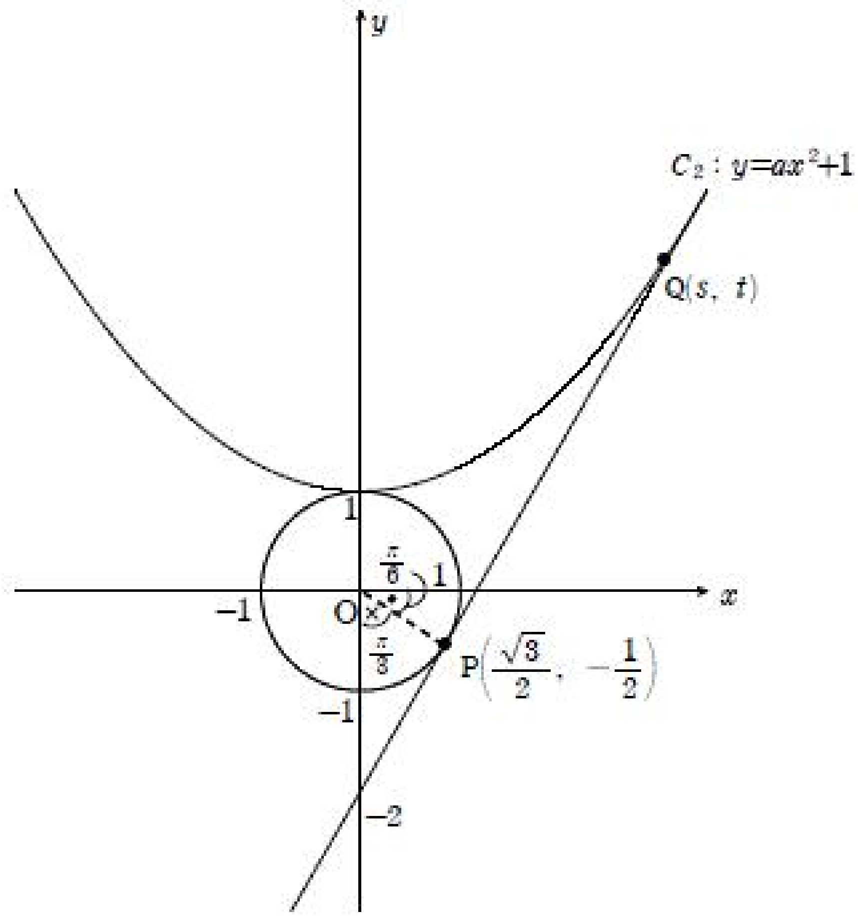
$$D=3-12a=0$$
$$\therefore \underline{a=\frac{1}{4}} \quad \cdots\cdots(答)$$

このとき①は，

$$\frac{1}{4}x^2-\sqrt{3}x+3=0$$
$$\Longleftrightarrow \frac{1}{4}\left(x-2\sqrt{3}\right)^2=0$$
$$\therefore x=2\sqrt{3}$$

したがって，

$$\underline{s=2\sqrt{3},\ t=4} \quad \cdots\cdots(答)$$



(2)　求める面積を S とする．

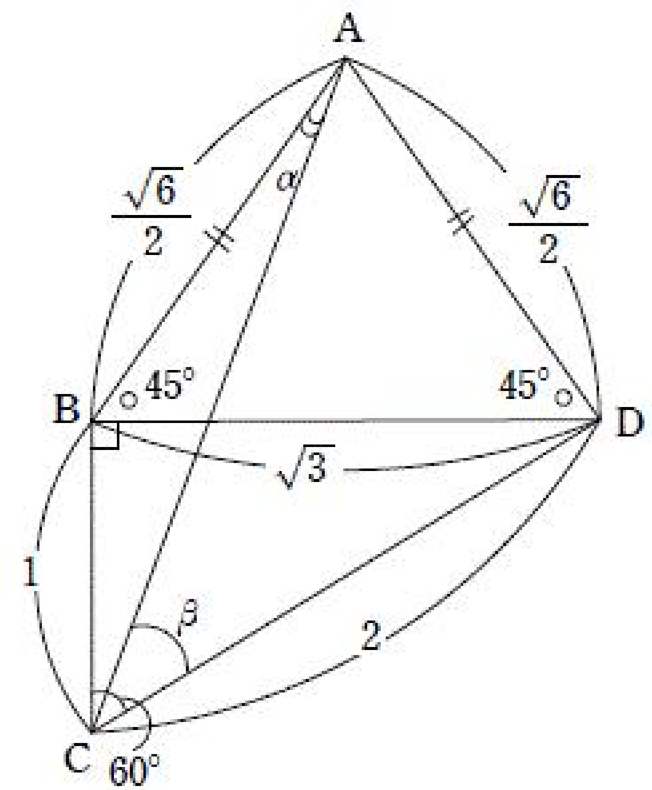
$$S=\int_0^{2\sqrt{3}}\frac{1}{4}\left(x-2\sqrt{3}\right)^2dx$$
$$=\frac{1}{12}\left[\left(x-2\sqrt{3}\right)^3\right]_0^{2\sqrt{3}}$$
$$=\underline{2\sqrt{3}} \quad \cdots\cdots(答)$$

(3)　求める面積を T とする．

$$T=2\sqrt{3}-\frac{1}{2}\cdot1\cdot\sqrt{3}-\frac{1}{2}\cdot1^2\cdot\frac{2}{3}\pi$$
$$=2\sqrt{3}-\frac{\sqrt{3}}{2}-\frac{\pi}{3}$$
$$=\underline{\frac{3\sqrt{3}}{2}-\frac{\pi}{3}} \quad \cdots\cdots(答)$$

[2]

(1) $\mathrm{BD}^2=\left(\sqrt{3}\right)^2=\underline{3}$ ……(答)



(2) 三角形 ABC で余弦定理より，

$$\mathrm{AC}^2=1^2+\left(\frac{\sqrt{6}}{2}\right)^2-2\cdot1\cdot\frac{\sqrt{6}}{2}\cos135^\circ$$

$$=1+\frac{6}{4}+\sqrt{6}\cdot\frac{1}{\sqrt{2}}$$

$$=\underline{\frac{5}{2}+\sqrt{3}}\ \text{……(答)}$$

(3) 三角形 ABC で余弦定理より，

$$\cos\alpha=\frac{\frac{6}{4}+\frac{5}{2}+\sqrt{3}-1}{2\cdot\frac{\sqrt{6}}{2}\cdot\sqrt{\frac{5}{2}+\sqrt{3}}}=\frac{3+\sqrt{3}}{\sqrt{6}\sqrt{\frac{5}{2}+\sqrt{3}}}$$

$$\therefore\cos^2\alpha=\frac{\left(3+\sqrt{3}\right)^2}{6\left(\frac{5}{2}+\sqrt{3}\right)}=\frac{4+2\sqrt{3}}{5+2\sqrt{3}}=\underline{\frac{8+2\sqrt{3}}{13}}\ \text{……(答)}$$

三角形 CDA で余弦定理より，

$$\cos\beta=\frac{4+\frac{5}{2}+\sqrt{3}-\frac{6}{4}}{2\cdot2\cdot\sqrt{\frac{5}{2}+\sqrt{3}}}=\frac{5+\sqrt{3}}{4\sqrt{\frac{5}{2}+\sqrt{3}}}$$

$$\therefore\cos^2\beta==\frac{28+10\sqrt{3}}{16\left(\frac{5}{2}+\sqrt{3}\right)}=\underline{\frac{40-3\sqrt{3}}{52}}\ \text{……(答)}$$

[3]

(1)

$$\overrightarrow{\mathrm{OA}}=(s,s,s)\quad(s>0)$$

$$\begin{aligned}\vec{d}&=\overrightarrow{\mathrm{OB}}-t\overrightarrow{\mathrm{OA}}\\&=(-1,1,1)-t(s,s,s)\\&=(-1-st,1-st,1-st)\end{aligned}$$

ここで，　$\overrightarrow{\mathrm{OA}}\perp\vec{d}$ より，

$$\overrightarrow{\mathrm{OA}}\cdot\vec{d}=s(-1-st)+s(1-st)+s(1-st)=0$$

$$\Longleftrightarrow 1-3st=0$$

したがって，

$$\underline{t=\frac{1}{3s}\quad(s>0)}\ \cdots\cdots(\text{答})$$

(2)　$\overrightarrow{\mathrm{OA}}\perp\vec{d}$ のとき，(1)より，

$$t=\frac{1}{3s}\quad(s>0)\cdots\cdots①$$

$\overrightarrow{\mathrm{OA}}\perp\vec{e}$ のとき，

$$\begin{aligned}\vec{e}&=\overrightarrow{\mathrm{OC}}-u\overrightarrow{\mathrm{OA}}-v\overrightarrow{\mathrm{OB}}\\&=(0,0,1)-u(s,s,s)-v(-1,1,1)\\&=(-su+v,-su-v,1-su-v)\end{aligned}$$

$$\overrightarrow{\mathrm{OA}}\cdot\vec{e}=s(-su+v)+s(-su-v)+s(1-su-v)=0\Longleftrightarrow 1-3su-v=0$$

$$\therefore 3su+v=1\cdots\cdots②$$

$\vec{d}\perp\vec{e}$ のとき，

$$\vec{d}\cdot\vec{e}=(-1-st)(-su+v)+(1-st)(-su-v)+(1-st)(1-su-v)=0$$

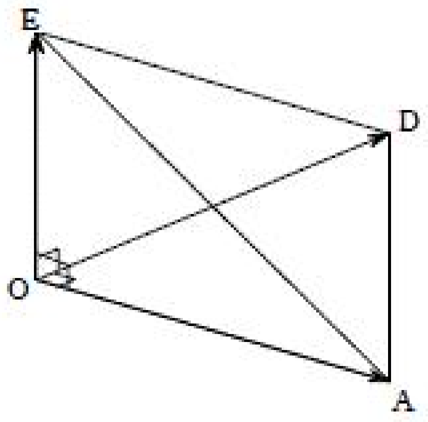
$$\Longleftrightarrow -4v+1=0\quad\left(①\text{より},\ s\,t=\frac{1}{3}\right)$$

したがって，

$$\left.\begin{aligned}&v=\frac{1}{4}\\&\underline{u=\frac{1}{4s}\ (②\text{より})}\end{aligned}\right.\ \cdots\cdots(\text{答})$$

(3)

$$\begin{cases}\overrightarrow{\mathrm{OA}}\perp\overrightarrow{\mathrm{OD}}\\\overrightarrow{\mathrm{OA}}\perp\overrightarrow{\mathrm{OE}}\\\overrightarrow{\mathrm{OD}}\perp\overrightarrow{\mathrm{OE}}\end{cases}$$ を全て満たす四面体 OADE は右図の通り



$$\left|\overrightarrow{\mathrm{OA}}\right|=\sqrt{s^2+s^2+s^2}=\sqrt{3}s$$

$$\overrightarrow{\mathrm{OD}}=\left(-\frac{4}{3},\frac{2}{3},\frac{2}{3}\right)\text{より},\ \left|\overrightarrow{\mathrm{OD}}\right|=\frac{2\sqrt{6}}{3}$$

$$\overrightarrow{\mathrm{OE}}=\left(0,-\frac{1}{2},\frac{1}{2}\right)\text{より},\ \left|\overrightarrow{\mathrm{OE}}\right|=\frac{\sqrt{2}}{2}$$

$$\therefore V=\left(\frac{1}{2}\cdot\sqrt{3}s\cdot\frac{2\sqrt{6}}{3}\right)\cdot\frac{\sqrt{2}}{2}\cdot\frac{1}{3}=2$$

したがって，

$$\underline{s=6}\ \cdots\cdots(\text{答})$$

[4]

(1)

$$4\text{回}\begin{cases}(\text{オ})\cdots\cdots 3\text{回}\\(\text{ウ})\cdots\cdots 1\text{回}\end{cases}$$

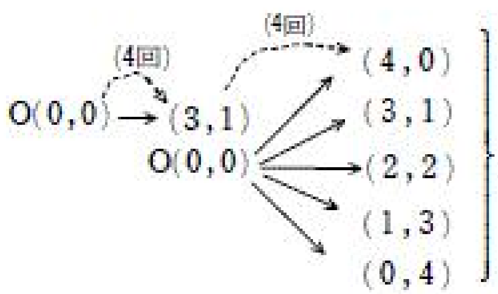
$${}_4C_3\left(\frac{1}{2}\right)^4=\frac{4}{16}=\underline{\frac{1}{4}}\quad\cdots\cdots(\text{答})$$

(2)

$$8\text{回}\begin{cases}(\text{オ})\cdots\cdots 5\text{回}\\(\text{ウ})\cdots\cdots 3\text{回}\end{cases}\text{のうち，}\quad\begin{cases}(\text{オ})\cdots\cdots 3\text{回}\\(\text{ウ})\cdots\cdots 1\text{回}\end{cases}\text{かつ}\begin{cases}(\text{オ})\cdots\cdots 2\text{回}\\(\text{ウ})\cdots\cdots 2\text{回}\end{cases}\text{を除いたもの}$$

$$\begin{aligned}&{}_8C_5\left(\frac{1}{2}\right)^8-\frac{1}{4}\cdot{}_4C_2\left(\frac{1}{2}\right)^4\\&=\frac{56}{256}-\frac{6}{64}\\&=\underline{\frac{1}{8}}\quad\cdots\cdots(\text{答})\end{aligned}$$

(3)



どの点も x, y 成分の和は 4 以下

ちなみに，途中(3,1)を通らないケースでは，x, y 成分の和はすべて 4 より大きくなる．

$$\frac{1}{4}\cdot 1=\underline{\frac{1}{4}}\quad\cdots\cdots(\text{答})$$

(4)

$$4n(\text{回})=\underbrace{4(n-k)(\text{回})}_{\substack{(n-k)\text{回}\\ O(0,0)\to(3,1)\\ \text{が起こる．}}}+\underbrace{4k(\text{回})}_{\substack{\text{任意}\\ \left(\substack{\text{どの点に移動しても}\\ x,\,y\text{ 成分の和は }4k\text{ 以下}}\right)}}$$

$$\left(\frac{1}{4}\right)^{n-k}\cdot 1^{4k}=\underline{\left(\frac{1}{4}\right)^{n-k}}\quad\cdots\cdots(\text{答})$$

[5]

(1) $$f(a)=\frac{1}{n}\sum_{k=1}^{n}\left(x_k-a\right)^2=a^2-\frac{2}{n}a\sum_{k=1}^{n}x_k+\frac{1}{n}\sum_{k=1}^{n}{x_k}^2$$

ここで，$m=\frac{1}{n}\sum_{k=1}^{n}x_k$ とおくと，

$$\begin{aligned} f(a)&=a^2-2am+\frac{1}{n}\sum_{k=1}^{n}{x_k}^2 \\ &=(a-m)^2-m^2+\frac{1}{n}\sum_{k=1}^{n}{x_k}^2 \end{aligned}$$

よって，$a=m$ のとき $f(a)$ は最小値 $f(m)=\frac{1}{n}\sum_{k=1}^{n}\left(x_k-m\right)^2$ をとり，

これは $x_1, x_2, x_3, \cdots\cdots, x_n, x_{n+1}$ の分散である．（証明終）

(2)

$$\begin{aligned} &\frac{1}{n}\sum_{k=1}^{n}\left(y_k-\overline{y_k}\right)^2>\frac{1}{n}\sum_{k=1}^{n}\left(z_k-\overline{z_k}\right)^2 \\ \Longleftrightarrow &\sum_{k=1}^{n}\left(y_k-\overline{y_k}\right)^2>\sum_{k=1}^{n}\left(cy_k-c\,\overline{y_k}\right)^2 \quad \left(y_k=k,\ z_k=ck\text{ より},\ z_k=cy_k\right) \\ \Longleftrightarrow &1>c^2 \end{aligned}$$

したがって，

$\underline{-1<c<1}$ ……（答）

(3) $x_1, x_2, x_3, \cdots\cdots, x_n, x_{n+1}$ の平均値を $\overline{x'}$ とおくと

$$\begin{aligned} \overline{x'}&=\frac{\left(x_1+x_2+x_3+\cdots\cdots+x_n\right)+x_{n+1}}{n+1} \\ &=\frac{x_1+x_2+x_3+\cdots\cdots+x_n}{n}\cdot\frac{n}{n+1}+\frac{x_{n+1}}{n+1} \\ &=\frac{n\overline{x}}{n+1}+\frac{x_{n+1}}{n+1} \\ &=\underline{\frac{n\overline{x}+x_{n+1}}{n+1}} \text{ ……（答）} \end{aligned}$$

(4)

（平均値）$\frac{40\cdot40+40}{41}=\frac{40(40+1)}{41}=\underline{40}$ ……（答）

（分散）$$\begin{aligned} \frac{1}{41}\sum_{k=1}^{41}\left(x_k-40\right)^2&=\frac{1}{41}\sum_{k=1}^{40}\left(x_k-40\right)^2+\frac{1}{41}\left(x_{41}-40\right)^2 \\ &=\frac{40}{41}\cdot\frac{1}{40}\sum_{k=1}^{40}\left(x_k-40\right)^2 \\ &=\frac{40}{41}\cdot670 \\ &=653.6\cdots \\ \therefore\ &\underline{654} \text{ ……（答）} \end{aligned}$$

（中央値）$\underline{40}$ ……（答）　←下記参照

10, 10, 10, 10, 10, 10, 10, 20, 20, 20, 20, 20, 20, 20, 30, 30, 30, 30, 30, 30, ▼40,
40, 40, 40, 40, 40, $\underset{\text{新たなデータ}}{\underline{40}}$, 50, 50, 50, 60, 60, 60, 60, 70, 70, 70, 70, 80, 100, 120