〔1〕

(1)

$$\sin\theta-\cos\theta>1$$

$$\iff \sqrt{2}\sin\left(\theta-\frac{\pi}{4}\right)>1$$

$$\iff \sin\left(\theta-\frac{\pi}{4}\right)>\frac{1}{\sqrt{2}}$$

ここで， $0\leqq\theta<2\pi$ より，

$$-\frac{\pi}{4}\leqq\theta-\frac{\pi}{4}<\frac{7}{4}\pi$$

よって，

$$\frac{\pi}{4}<\theta-\frac{\pi}{4}<\frac{3}{4}\pi \qquad \therefore\ \underline{\frac{\pi}{2}<\theta<\pi}\ (答)$$

(2) $t=\sin\theta\cos\theta=\dfrac{1}{2}\sin2\theta\ \ (\pi<2\theta<2\pi)$

よって，

$$-1\leqq\sin2\theta<0 \qquad \therefore\ \underline{-\frac{1}{2}\leqq t<0}\ (答)$$

(3)

$$\begin{aligned}L^2&=(\sin\theta-\cos\theta)^2+(\sin^2\theta-\cos^2\theta)^2\\&=1-2\sin\theta\cos\theta+(\sin^2\theta+\cos^2\theta)^2-4\sin^2\theta\cos^2\theta\\&=-4t^2-2t+2\end{aligned}$$

ここで， $L>0$ より，

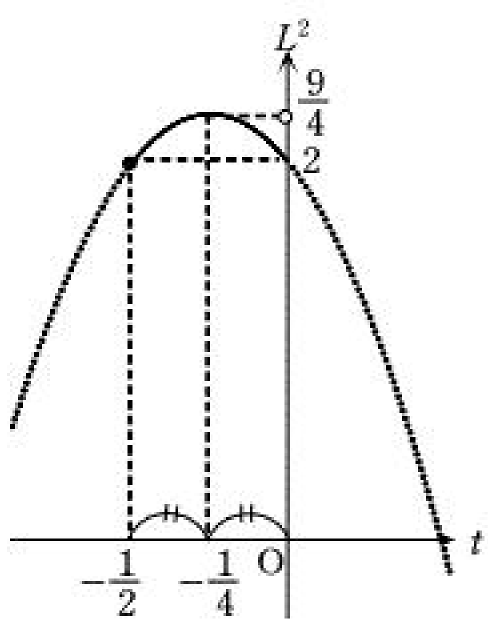
$$\underline{L=\sqrt{-4t^2-2t+2}\ \left(-\frac{1}{2}\leqq t<0\right)}\ (答)$$

(4)

$$L^2=-4\left(t+\frac{1}{4}\right)^2+\frac{9}{4}$$

$$\therefore\ 2\leqq L^2\leqq\frac{9}{4}$$

$$\therefore\ \begin{cases}(最大値)=\dfrac{3}{2} & \left(t=-\dfrac{1}{4}\text{ より}, \theta=\dfrac{7}{12}\pi, \dfrac{11}{12}\pi\right)\\(最小値)=\sqrt{2} & \left(t=-\dfrac{1}{2}\text{ より}, \theta=\dfrac{3}{4}\pi\right)\end{cases} \qquad (答)$$



〔2〕

(1)　点 $\mathrm{P}(t,0)$ を通り，傾き 1 の直線は，

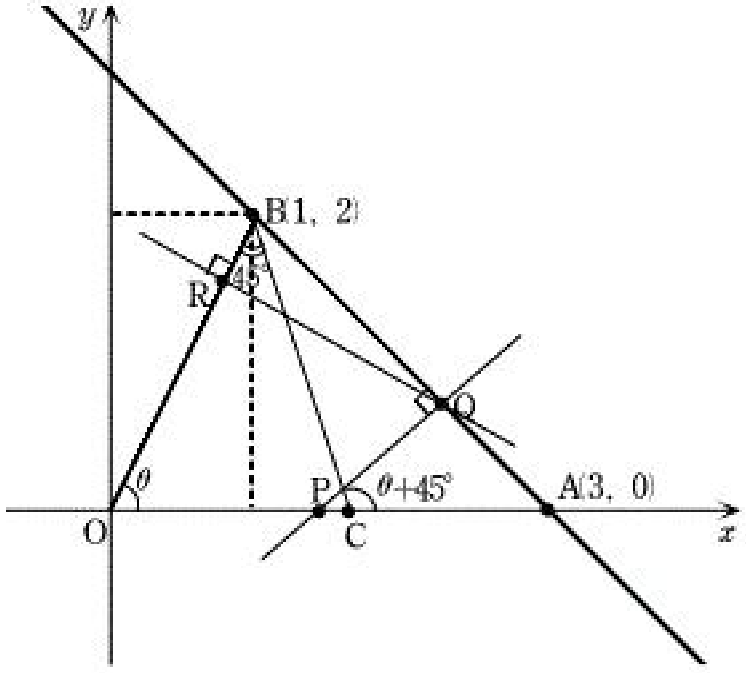
$$y=x-t$$

直線 AB は，

$$y=-(x-3) \iff y=-x+3$$

よって，

$$\underline{\text{点 Q}\left(\frac{t+3}{2},\ \frac{-t+3}{2}\right)\ （答）}$$



(2)　直線 QR は，

$$y=-\frac{1}{2}\left(x-\frac{t+3}{2}\right)+\frac{-t+3}{2}$$

$$\iff \quad y=-\frac{1}{2}x+\frac{9-t}{4}$$

直線 OB は，

$$y=2x$$

よって，

$$\underline{\text{点 R}\left(\frac{-t+9}{10},\ \frac{-t+9}{5}\right)\ （答）}$$

(3)　$\angle\mathrm{AOB}=\theta$ とおくと，　　$\tan\theta=2$

ここで，　$\angle\mathrm{ACB}=\theta+45^\circ$ より，

$$\begin{aligned}(\text{直線 BC の傾き})&=\tan(\theta+45^\circ)\\&=\frac{2+1}{1-2\cdot 1}\\&=-3\end{aligned}$$

直線 RP′ は，$y=-3\left(x-\dfrac{-t+9}{10}\right)+\dfrac{-t+9}{5}$

よって，

$$\underline{\text{点 P}'\left(\frac{-t+9}{6},\ 0\right)\ （答）}$$

(4)

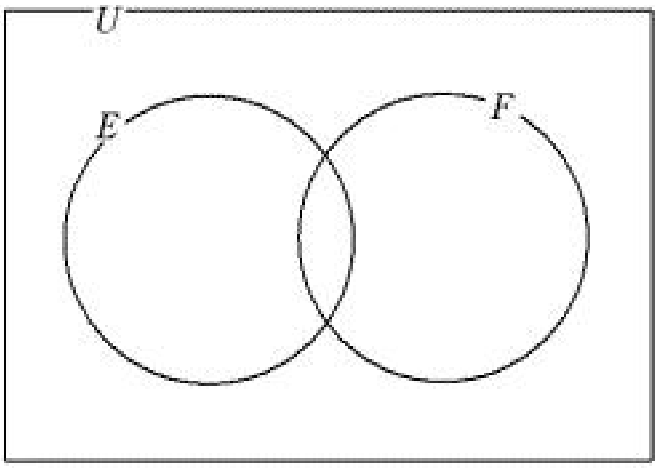
$$\begin{aligned}t_{n+1}&=\frac{-t_n+9}{6}\\&=-\frac{1}{6}t_n+\frac{3}{2}\ \ (t_1=2)\end{aligned}$$

$$(\text{与式}) \iff \quad t_{n+1}-\frac{9}{7}=-\frac{1}{6}\left(t_n-\frac{9}{7}\right)$$

$$\therefore\ t_n-\frac{9}{7}=\left(2-\frac{9}{7}\right)\left(-\frac{1}{6}\right)^{n-1} \qquad \therefore\ \underline{t_n=\frac{5}{7}\left(-\frac{1}{6}\right)^{n-1}+\frac{9}{7}\ \ (n=1,2,3,\cdots\cdots)\ \ （答）}$$

〔3〕 「Xが5の倍数」＝事象E

「Xが3の倍数」＝事象F



(1)

$$P(E)=1-P\left(\overline{E}\right)$$
$$=\underline{1-\left(\frac{5}{6}\right)^n}\ (\text{答})$$

(2)

$$P(E)>0.99$$
$$\Longleftrightarrow\quad 1-\left(\frac{5}{6}\right)^n>\frac{99}{100}$$
$$\Longleftrightarrow\quad \frac{1}{100}>\left(\frac{5}{6}\right)^n$$
$$\Longleftrightarrow\quad \log_2 10^{-2}>n\left(\log_2 5-\log_2 2\cdot 3\right)$$
$$\Longleftrightarrow\quad -2(1+2.322)>n(2.322-1-1.585)$$
$$\Longleftrightarrow\quad n>\frac{6.644}{0.263}(=25.2\cdots\cdots)$$
$$\therefore\ \underline{\left(\text{最小の}\ n\right)=26\ (\text{答})}$$

(3)

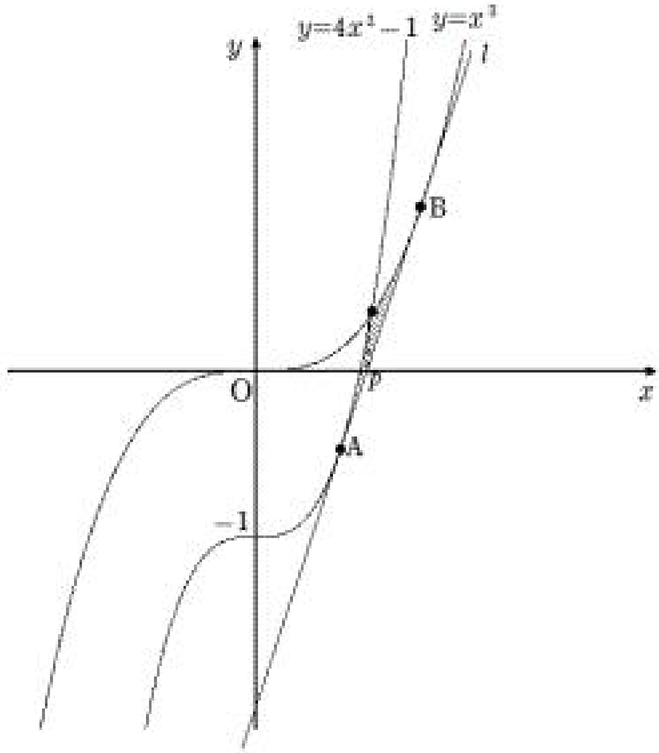
$$P\underbrace{\left(\overline{E}\cap\overline{F}\right)}_{\text{1 or 2 or 4}}=\left(\frac{3}{6}\right)^n=\underline{\left(\frac{1}{2}\right)^n}\qquad(\text{答})$$

(4)

$$P(E\cap F)=1-P\left(\overline{E\cap F}\right)$$
$$=1-P\left(\overline{E}\cup\overline{F}\right)$$
$$=1-\left\{P\left(\overline{E}\right)+P\left(\overline{F}\right)-\underline{\underline{P\left(\overline{E}\cap\overline{F}\right)}}\right\}$$
$$=1-\left\{\left(\frac{5}{6}\right)^n+\left(\frac{4}{6}\right)^n-\left(\frac{1}{2}\right)^n\right\}$$
$$=\underline{1-\left(\frac{5}{6}\right)^n-\left(\frac{2}{3}\right)^n+\left(\frac{1}{2}\right)^n}\quad(\text{答})$$

〔4〕

(1)



$$4x^3-1=x^3 \iff x^3=\frac{1}{3}$$

$$\therefore\ \underline{p=\frac{1}{\sqrt[3]{3}} \qquad \text{(答)}}$$

(2) $\mathrm{A}\left(a,4a^3-1\right)$, $y'=12x^2$ より,

$$l: y=12a^2(x-a)+4a^3-1$$

$$\iff \underline{y=12a^2x-8a^3-1 \qquad \text{(答)}}$$

(3) $y=x^3$ 上の点 $\left(b,b^3\right)$ での接線は,

$$y=3b^2(x-b)+b^3$$

$$\iff y=3b^2x-2b^3$$

これを l と同じと考えて,

$$\begin{cases}12a^2=3b^2 & (b=2a)\\ -8a^3-1=-2b^3\end{cases}$$

よって,

$$-8a^3-1=-2(2a)^3$$

$$\iff 8a^3=1$$

$$\iff a^3=\frac{1}{8} \qquad \therefore\ \underline{a=\frac{1}{2}\ (>0)\ \left(b=1, l\text{ は } y=3x-2\right) \qquad \text{(答)}}$$

(4)

$$S=\int_{\frac{1}{2}}^{p}\left\{\left(4x^3-1\right)-(3x-2)\right\}dx+\int_{p}^{1}\left\{x^3-(3x-2)\right\}dx$$

$$=\left[x^4-\frac{3}{2}x^2+x\right]_{\frac{1}{2}}^{p}+\left[\frac{x^4}{4}-\frac{3}{2}x^2+2x\right]_{p}^{1}$$

$$=\frac{3}{4}p^4-p+\frac{9}{16}$$

$$\left(p=\frac{1}{\sqrt[3]{3}}\right)$$

$$\underline{=-\frac{3}{4\sqrt[3]{3}}+\frac{9}{16} \qquad \text{(答)}}$$