$\boxed{\text{I}}$

(1)
$$\begin{pmatrix}1&-2\\1&-1\end{pmatrix}^{-1}=\frac{1}{-1-(-2)}\begin{pmatrix}-1&2\\-1&1\end{pmatrix}=\begin{pmatrix}\overset{\text{アイ}}{\boxed{-1}}&\overset{\text{ウ}}{\boxed{2}}\\\underset{\text{エオ}}{\boxed{-1}}&\underset{\text{カ}}{\boxed{1}}\end{pmatrix}\qquad\text{(答)}$$

$A=\begin{pmatrix}a&b\\c&d\end{pmatrix}$ （a,b,c,d は実数）とおく.

$$\begin{cases}\begin{pmatrix}1&-2\\2&-4\end{pmatrix}\begin{pmatrix}a&b\\c&d\end{pmatrix}=\begin{pmatrix}a-2c&b-2d\\2a-4c&2b-4d\end{pmatrix}=\begin{pmatrix}-11&1\\-22&2\end{pmatrix}\\\begin{pmatrix}2&-2\\3&-3\end{pmatrix}\begin{pmatrix}a&b\\c&d\end{pmatrix}=\begin{pmatrix}2a-2c&2b-2d\\3a-3c&3b-3d\end{pmatrix}=\begin{pmatrix}-8&6\\-12&9\end{pmatrix}\end{cases}$$

これより,

$$\begin{cases}a-2c=-11\\b-2d=1\\a-c=-4\\b-d=3\end{cases}$$

$$\iff\begin{pmatrix}1&-2\\1&-1\end{pmatrix}\underset{\overset{\|}{A}}{\underline{\begin{pmatrix}a&b\\c&d\end{pmatrix}}}=\begin{pmatrix}-11&1\\-4&3\end{pmatrix}$$

$$\iff A=\begin{pmatrix}1&-2\\1&-1\end{pmatrix}^{-1}\begin{pmatrix}-11&1\\-4&3\end{pmatrix}$$
$$=\begin{pmatrix}-1&2\\-1&1\end{pmatrix}\begin{pmatrix}-11&1\\-4&3\end{pmatrix}$$
$$=\begin{pmatrix}\overset{\text{キ}}{\boxed{3}}&\overset{\text{ク}}{\boxed{5}}\\\underset{\text{ケ}}{\boxed{7}}&\underset{\text{コ}}{\boxed{2}}\end{pmatrix}\qquad\text{(答)}$$

(2) $y=\sqrt{x}\ \left(y=x^{\frac{1}{2}}\right)$ より, $\quad y'=\frac{1}{2}x^{-\frac{1}{2}}$

$$a=\frac{\overset{\text{ア}}{\boxed{1}}}{\underset{\text{イ}}{\boxed{2}}},\ b=\frac{\overset{\text{ウエ}}{\boxed{-1}}}{\underset{\text{オ}}{\boxed{2}}}\qquad\text{(答)}$$

(1, 1)での接線は,

$$y=\frac{1}{2}(x-1)+1$$
$$\iff y=\frac{\overset{\text{カ}}{\boxed{1}}}{\underset{\text{キ}}{\boxed{2}}}x+\frac{\overset{\text{ク}}{\boxed{1}}}{\underset{\text{ケ}}{\boxed{2}}}\qquad\text{(答)}$$

$x=1+h$ を代入して,

$$y=\frac{1}{2}(1+h)+\frac{1}{2}=\overset{\text{コ}}{\boxed{1}}+\frac{\overset{\text{サ}}{\boxed{1}}}{\underset{\text{シ}}{\boxed{2}}}h\qquad\text{(答)}$$

$$\sqrt{17}=\sqrt{16+1}=\sqrt{16\left(1+\frac{1}{16}\right)}=4\sqrt{1+\frac{\overset{\text{ス}}{\boxed{1}}}{\underset{\text{セソ}}{\boxed{16}}}}\qquad\text{(答)}$$

この近似値は,

$$4\left(1+\frac{1}{2}\cdot\frac{1}{16}\right)=\overset{\text{タ}}{\boxed{4}}\left(1+\frac{\overset{\text{チ}}{\boxed{1}}}{\underset{\text{ツテ}}{\boxed{32}}}\right)=\overset{\text{ト}}{\boxed{4}}+\frac{\overset{\text{ナ}}{\boxed{1}}}{\underset{\text{ニ}}{\boxed{8}}}\qquad\text{(答)}$$

$\sqrt{17}$ は $4+\frac{1}{8}=4.125$ よりやや小さいことから,

$$\overset{\text{ヌ}}{\boxed{4}}.\overset{\text{ネノ}}{\boxed{12}}\qquad\text{(答)}$$

$$\sqrt{37}=\sqrt{36+1}=\sqrt{36\left(1+\frac{1}{36}\right)}=6\sqrt{1+\frac{1}{36}}$$

この近似値は,

$$6\left(1+\frac{1}{2}\cdot\frac{1}{36}\right)=6\left(1+\frac{1}{72}\right)=\overset{\text{ハ}}{\boxed{6}}+\frac{\overset{\text{ヒ}}{\boxed{1}}}{\underset{\text{フヘ}}{\boxed{12}}}\qquad\text{(答)}$$

$\sqrt{37}$ は $6+\frac{1}{12}=6.0833\cdots$ よりやや小さいことから,

$$\overset{\text{ホ}}{\boxed{6}}.\overset{\text{マミ}}{\boxed{08}}\qquad\text{(答)}$$

(3) $y=x^3-4x$ を x 軸方向に p, y 軸方向に q だけ平行移動

$$y-q=(x-p)^3-4(x-p)$$
$$\iff y=x^3-3px^2+\left(3p^2-4\right)x-p^3+4p+q\quad\cdots\cdots①$$

これが,

$$y=(x-1)(x-2)(x-\alpha)$$
$$=(x^2-3x+2)(x-\alpha)$$
$$=x^3-(\alpha+3)x^2+(3\alpha+2)x-2\alpha\quad\cdots\cdots②$$

となる.

①と②より,

$$\begin{cases}3p=\alpha+3&\cdots\cdots③\\3p^2-4=3\alpha+2&\cdots\cdots④\\-p^3+4p+q=-2\alpha&\cdots\cdots⑤\end{cases}\qquad\text{(答)}$$

③, ④より,

$$p^2-3p+1=0\qquad\therefore\ p=\frac{3\pm\sqrt{5}}{2}$$

・$p=\dfrac{\overset{\text{ア}}{\boxed{3}}-\sqrt{\overset{\text{イ}}{\boxed{5}}}}{\underset{\text{ウ}}{\boxed{2}}}$ のとき,（3つ目の交点の x 座標）$=\alpha=\dfrac{\overset{\text{オ}}{\boxed{3}}-\overset{\text{カ}}{\boxed{3}}\sqrt{\overset{\text{キ}}{\boxed{5}}}}{\underset{\text{ク}}{\boxed{2}}}$

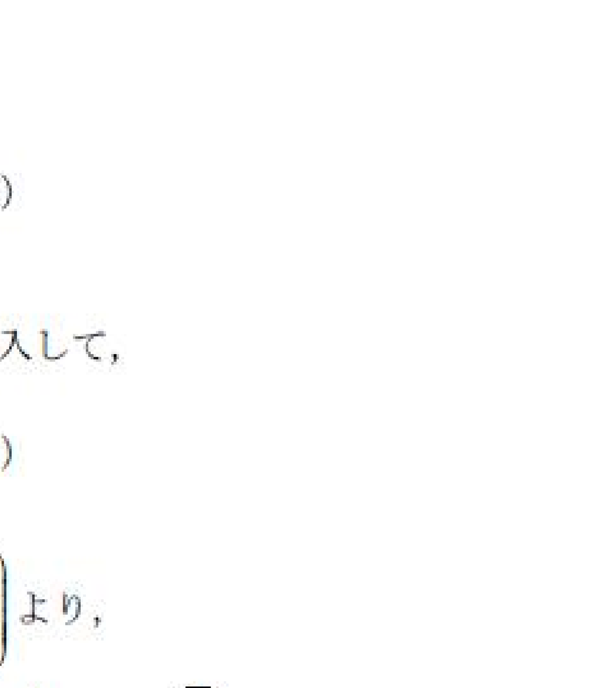
⑤より, $q=\sqrt{\overset{\text{エ}}{\boxed{5}}}$ （答）

・$p=\dfrac{\overset{\text{ケ}}{\boxed{3}}+\sqrt{\overset{\text{コ}}{\boxed{5}}}}{\underset{\text{サ}}{\boxed{2}}}$ のとき,（3つ目の交点の x 座標）$=\alpha=\dfrac{\overset{\text{ス}}{\boxed{3}}+\overset{\text{セ}}{\boxed{3}}\sqrt{\overset{\text{ソ}}{\boxed{5}}}}{\underset{\text{タ}}{\boxed{2}}}$

⑤より, $q=-\sqrt{\overset{\text{シ}}{\boxed{5}}}$ （答）

(4) xy 座標で考えてみる.

$\mathrm{A_0}\left(\sqrt{2},0\right),\mathrm{B_0}\left(0,\sqrt{2}\right)$ より, $\mathrm{A_1}\left(\frac{3\sqrt{2}+\sqrt{6}}{6},\frac{3\sqrt{2}-\sqrt{6}}{6}\right)$



$\mathrm{OA_1}=\frac{2\sqrt{3}}{3}$ より, $\frac{\mathrm{A_1O}}{\mathrm{A_0O}}=\frac{2\sqrt{3}}{3\sqrt{2}}=\frac{\sqrt{6}}{3}$

正方形 $\mathrm{A_0B_0C_0D_0}$, 正方形 $\mathrm{A_1B_1C_1D_1}$, … はすべて相似.（相似な図形の中心は O）

$$\therefore\ \frac{\mathrm{A}_n\mathrm{O}}{\mathrm{A}_{n-1}\mathrm{O}}=\frac{\sqrt{\overset{\text{ア}}{\boxed{6}}}}{\underset{\text{イ}}{\boxed{3}}}\qquad\text{(答)}$$

$\mathrm{A}_n\mathrm{O}=\mathrm{OA}_n=\sqrt{2}\left(\frac{\sqrt{6}}{3}\right)^n$ より, $\mathrm{OB}_n=\sqrt{2}\left(\frac{\sqrt{6}}{3}\right)^n$

$$\mathrm{A}_n\mathrm{B}_n=\sqrt{2}\cdot\sqrt{2}\left(\frac{\sqrt{6}}{3}\right)^n=\overset{\text{ウ}}{\boxed{2}}\left(\frac{\sqrt{\overset{\text{エ}}{\boxed{6}}}}{\underset{\text{オ}}{\boxed{3}}}\right)^n\qquad\text{(答)}$$

次に極座標を使い, $\mathrm{A_0}\left(\sqrt{2},0\right)$ だから, $r=ae^{b\theta}$ へ代入して,

$$\sqrt{2}=ae^0\qquad\therefore\ a=\sqrt{\overset{\text{カ}}{\boxed{2}}}\qquad\text{(答)}$$

さらに,

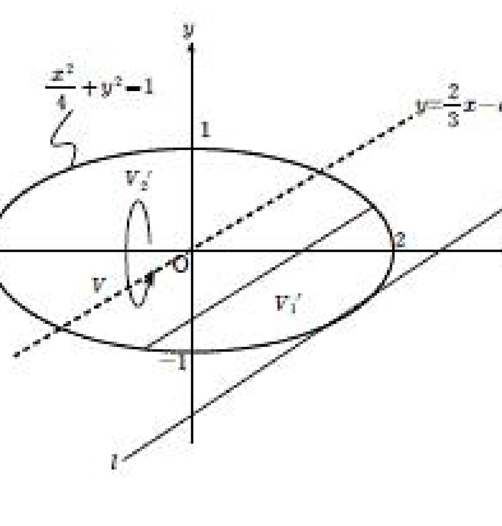
$$\mathrm{A_1}\left(\frac{2\sqrt{3}}{3},\frac{\pi}{12}\right)\leftarrow\left(\tan\frac{\pi}{12}=2-\sqrt{3}=\frac{3\sqrt{2}-\sqrt{6}}{3\sqrt{2}+\sqrt{6}}\right)\text{より,}$$

$$\frac{2\sqrt{3}}{3}=\sqrt{2}e^{\frac{\pi}{12}b}\iff e^{\frac{\pi}{12}b}=\frac{\sqrt{6}}{3}\iff\frac{\pi b}{12}=\log\frac{\sqrt{6}}{3}$$
$$=\log\left(\frac{6}{9}\right)^{\frac{1}{2}}$$
$$=\frac{1}{2}\log\frac{2}{3}$$

$$\therefore\ b=\frac{6}{\pi}\log\frac{\overset{\text{キ}}{\boxed{2}}}{\underset{\text{ク}}{\boxed{3}}}\qquad\text{(答)}$$

$$\left(\sqrt{2}\mathrm{OA_0}\right)^2+\left(\sqrt{2}\mathrm{OA_1}\right)^2+\left(\sqrt{2}\mathrm{OA_2}\right)^2+\cdots\cdots$$
$$=2\left(\mathrm{OA_0}^2+\mathrm{OA_1}^2+\mathrm{OA_2}^2+\cdots\cdots\qquad\right)\leftarrow\left(\mathrm{OA}_n{}^2=\frac{2}{3}\mathrm{OA}_{n-1}{}^2\right)$$
$$=2\cdot\frac{2}{1-\frac{2}{3}}$$
$$=\overset{\text{ケコ}}{\boxed{12}}\qquad\text{(答)}$$

(5)

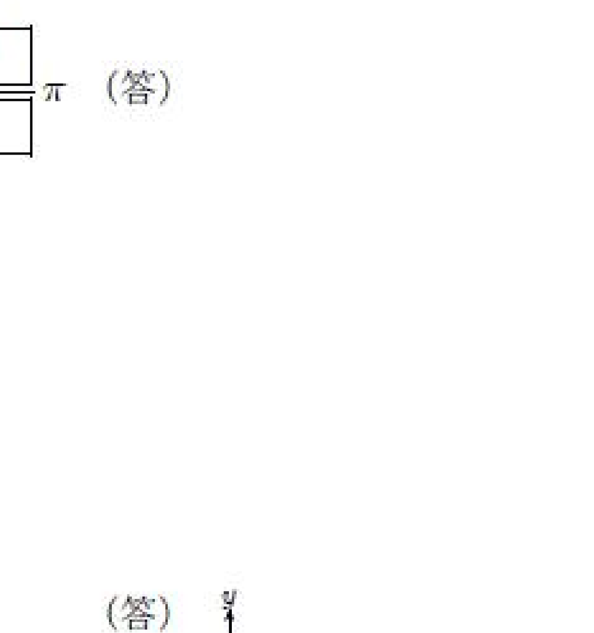


$$V_1=\pi\int_{\frac{1}{2}}^{1}(1-x^2)\,dx=\pi\left[x-\frac{x^3}{3}\right]_{\frac{1}{2}}^{1}=\frac{5}{24}\pi$$

$$V_2=\frac{4}{3}\pi\cdot1^3-\frac{5}{24}\pi=\frac{27}{24}\pi$$

$$\therefore\ V_1:V_2=\frac{5}{24}\pi:\frac{27}{24}\pi=\overset{\text{ア}}{\boxed{5}}:\overset{\text{イウ}}{\boxed{27}}\qquad\text{(答)}$$

$l:y=\frac{2}{3}x+k$ （$k<0$）とおく.



$$\frac{x^2}{4}+\left(\frac{2}{3}x+k\right)^2=1$$
$$\iff\ 25x^2+48kx+36(k^2-1)=0$$
$$D/4=(24k)^2-25\cdot36(k^2-1)$$
$$\iff\ k^2=\frac{25}{9}\ \therefore\ k=-\frac{5}{3}\ (<0)$$

$$l:y=\frac{2}{3}x-\frac{\overset{\text{エ}}{\boxed{5}}}{\underset{\text{オ}}{\boxed{3}}}\qquad\text{(答)}$$

$$（V\text{ の体積}）=\pi\int_{-2}^{2}\left(1-\frac{x^2}{4}\right)dx=2\pi\left[x-\frac{x^3}{12}\right]_0^2=\frac{\overset{\text{カ}}{\boxed{8}}}{\underset{\text{キ}}{\boxed{3}}}\pi\qquad\text{(答)}$$

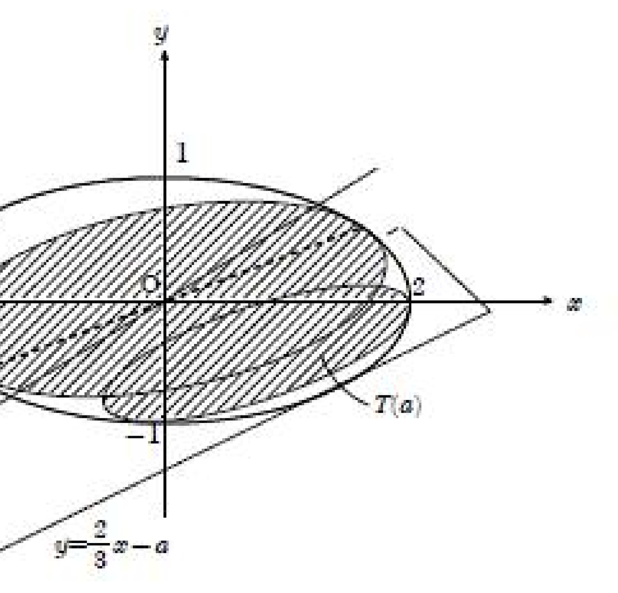
$a=0$ のとき($k=0$ のとき)

$$25x^2=36\qquad\therefore\ x=\pm\frac{6}{5}$$

（長軸の長さ）$=\frac{12}{5}\cdot\frac{\sqrt{13}}{3}=\frac{4\sqrt{13}}{5}$

（短軸の長さ）$=2$

$$\therefore\quad（\text{楕円の面積}）=\pi\cdot\frac{2\sqrt{13}}{5}\cdot1=\frac{\overset{\text{ク}}{\boxed{2}}\sqrt{\overset{\text{ケコ}}{\boxed{13}}}}{\underset{\text{サ}}{\boxed{5}}}\pi\qquad\text{(答)}$$



$a=\dfrac{\overset{\text{シ}}{\boxed{5}}}{\underset{\text{ス}}{\boxed{6}}}$ のときに V の体積は 5:27 に分割される.

＊ 厳密に解答すると次ページの通り.

（別解） 楕円 $\frac{x^2}{4}+y^2=1$ の x 軸回転させた

回転曲面は $\frac{x^2}{4}+y^2+z^2=1$ ……①

平面 $y=\frac{2}{3}x-a$ （$a\geqq0$） ……②

①, ②より,

$$\frac{x^2}{4}+\left(\frac{2}{3}x-a\right)^2+z^2=1$$
$$\iff\ \frac{x^2}{4}+\frac{4}{9}x^2-\frac{4a}{3}x+a^2+z^2=1$$
$$\iff\ \frac{\left(x-\frac{24}{25}a\right)^2}{\frac{36}{25}\left(1-\frac{9}{25}a^2\right)}+\frac{z^2}{1-\frac{9}{25}a^2}=1$$

この楕円の面積を $S(a)$ とすると,

（xz 平面に正射影したもの）

$$S(a)=\pi\cdot\frac{6}{5}\sqrt{1-\frac{9}{25}a^2}\cdot\sqrt{1-\frac{9}{25}a^2}$$
$$=\frac{6}{5}\left(1-\frac{9}{25}a^2\right)\pi$$

よって,

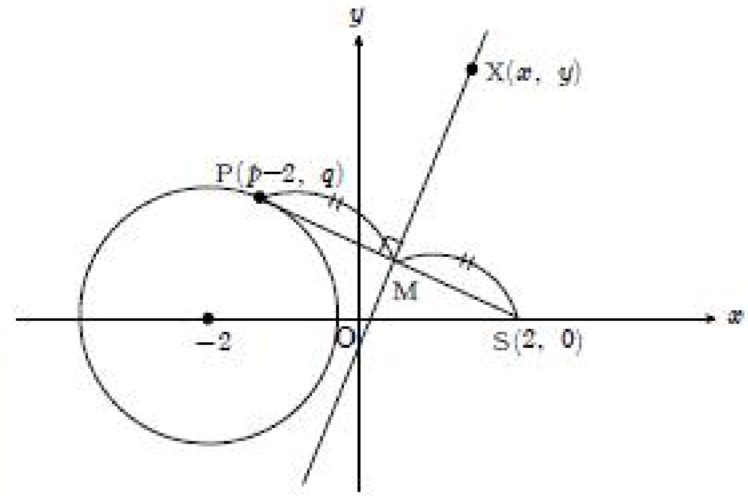
$$T(a)=\frac{\sqrt{13}}{3}\cdot\frac{6}{5}\left(1-\frac{9}{25}a^2\right)\pi$$
$$=\frac{2\sqrt{13}}{5}\pi\left(1-\frac{9}{25}a^2\right)\quad\left(0\leqq a\leqq\frac{5}{3}\right)$$

（小さい方の立体の体積）$=\frac{8}{3}\pi\cdot\frac{5}{5+27}=\frac{5}{12}\pi$ より,

$$\int_{a_0}^{\frac{5}{3}}\frac{2\sqrt{13}}{5}\pi\left(1-\frac{9}{25}a^2\right)\cdot\frac{3}{\sqrt{13}}da=\frac{6}{5}\pi\left[a-\frac{3}{25}a^3\right]_{a_0}^{\frac{5}{3}}$$
$$=\frac{6}{5}\pi\left(\frac{5}{3}-\frac{5}{9}-a_0+\frac{3}{25}a_0{}^3\right)=\frac{5}{12}\pi$$

$$\therefore\ \frac{3}{25}a_0{}^3-a_0+\frac{55}{72}=0\qquad\therefore\ a_0=a=\frac{\overset{\text{シ}}{\boxed{5}}}{\underset{\text{ス}}{\boxed{6}}}\qquad\text{(答)}$$

II



$$M\left(\frac{p}{2}, \frac{q}{2}\right)$$

$\overrightarrow{MX}=\left(x-\frac{p}{2}, y-\frac{q}{2}\right)$, $\overrightarrow{PS}=(4-p, -q)$ より，

$$\overrightarrow{MX}\cdot\overrightarrow{PS}=\left(x-\frac{p}{2}\right)(4-p)-q\left(y-\frac{q}{2}\right)=0$$

$$\Longleftrightarrow\ qy=-p\left(x+\overset{ア}{\boxed{2}}\right)+\left(\overset{イ}{\boxed{4}}\,x+\frac{r^2}{\underset{ウ}{\boxed{2}}}\right)$$

$$\left(\because\ \frac{p^2}{2}+\frac{q^2}{2}=\frac{r^2}{2}\right)\ （答）$$

$$\therefore\ q^2y^2=p^2(x+2)^2+\left(4x+\frac{r^2}{2}\right)^2-2p(x+2)\left(4x+\frac{r^2}{2}\right)$$

$$\Longleftrightarrow\ p^2(x+2)^2-2p(x+2)\left(4x+\frac{r^2}{2}\right)+\left(4x+\frac{r^2}{2}\right)^2-(r^2-p^2)y^2=0$$

$$\Longleftrightarrow\ p^2\{(x+2)^2+y^2\}-2p(x+2)\left(4x+\frac{r^2}{2}\right)+\left(4x+\frac{r^2}{2}\right)^2-r^2y^2=0$$

$$D/4=(x+2)^2\left(4x+\frac{r^2}{2}\right)^2-\{(x+2)^2+y^2\}\left\{\left(4x+\frac{r^2}{2}\right)^2-r^2y^2\right\}>0$$

$$\therefore\ r^2y^2+\left(r^2-\overset{エオ}{\boxed{16}}\right)x^2-\frac{r^2}{4}\left(r^2-\overset{エオ}{\boxed{16}}\right)=0\ （答）$$

ここで，

$$r^2y^2+(r^2-16)x^2-\frac{r^2}{4}(r^2-16)=0$$

$$\Longleftrightarrow\ (r^2-16)x^2+r^2y^2=\frac{r^2}{4}(r^2-16)$$

$$\Longleftrightarrow\ \frac{x^2}{\frac{r^2}{4}}+\frac{y^2}{\frac{r^2-16}{4}}=1$$ 焦点は $\left(\overset{カキ}{\boxed{-2}}, 0\right)$, $\left(\overset{ク}{\boxed{2}}, 0\right)$ （答）

・$r>\overset{ケ}{\boxed{4}}$ のとき　$\frac{r^2-16}{4}>0$ より　楕円 $\overset{コ}{\boxed{a}}$

・$r<\overset{ケ}{\boxed{4}}$ のとき　$\frac{r^2-16}{4}<0$ より　双曲線 $\overset{サ}{\boxed{c}}$

$(0<r<4)$

$\boxed{\text{III}}$

(1)　三角形の 3 つの内角の二等分線の交点を，その三角形の内心という．(答)

(2)

$$\begin{aligned}\vec{u}\cdot\vec{w}&=\vec{u}\cdot\left\{\vec{v}-\left(\vec{v}\cdot\vec{u}\right)\vec{u}\right\}\\&=\vec{u}\cdot\vec{v}-\left(\vec{u}\cdot\vec{v}\right)\left|\vec{u}\right|^2\\&=\vec{u}\cdot\vec{v}-\vec{u}\cdot\vec{v}\qquad\left(\because\ \left|\vec{u}\right|^2=1\right)\\&=0\end{aligned}$$

より，$\overrightarrow{u}\perp\overrightarrow{w}$（証明終）

(3)　$\left|\vec{u}\right|=\left|\vec{v}\right|=1,\ \vec{u}\not\parallel\vec{v}$ ……①

$\overrightarrow{w}=k\overrightarrow{v}+l\overrightarrow{u}$（$k>0,\ l>0$）に対して，

$$\overrightarrow{w}\cdot\overrightarrow{u}=k\overrightarrow{u}\cdot\overrightarrow{v}+l,\quad \overrightarrow{w}\cdot\overrightarrow{v}=l\overrightarrow{u}\cdot\overrightarrow{v}+k$$

よって，

$$\begin{aligned}\vec{x}&=k\vec{v}+l\vec{u}-\left(k\vec{u}\cdot\vec{v}+l\right)\vec{u}\\&=k\vec{v}-k\left(\vec{u}\cdot\vec{v}\right)\vec{u}\\&=k\left\{\vec{v}-\left(\vec{u}\cdot\vec{v}\right)\vec{u}\right\}\\\vec{y}&=k\vec{v}+l\vec{u}-\left(l\vec{u}\cdot\vec{v}+k\right)\vec{v}\\&=l\vec{u}-l\left(\vec{u}\cdot\vec{v}\right)\vec{v}\\&=l\left\{\vec{u}-\left(\vec{u}\cdot\vec{v}\right)\vec{v}\right\}\end{aligned}$$

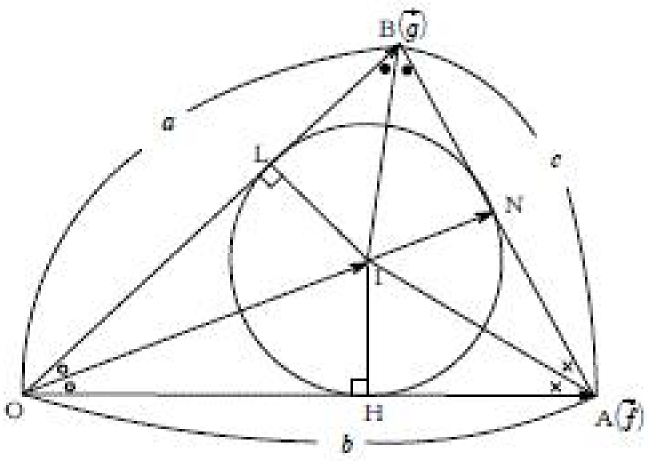
ここで，

$$\begin{aligned}&\left|\vec{x}\right|^2=\left|\vec{y}\right|^2\\\iff&k^2\left\{1+\left(\vec{u}\cdot\vec{v}\right)^2-2\left(\vec{u}\cdot\vec{v}\right)^2\right\}=l^2\left\{1+\left(\vec{u}\cdot\vec{v}\right)^2-2\left(\vec{u}\cdot\vec{v}\right)^2\right\}\\\iff&\left\{1-\left(\vec{u}\cdot\vec{v}\right)^2\right\}\left(k^2-l^2\right)=0\end{aligned}$$

①より，$\overrightarrow{u}\cdot\overrightarrow{v}\neq\pm1$　$\therefore\ k=l$（答）

(4)　内接円と辺 OA，辺 OB の接点を H，L とする．

(2)，(3)を用いて，



$$\begin{aligned}\overrightarrow{\mathrm{HI}}&=\overrightarrow{\mathrm{OI}}-\overrightarrow{\mathrm{OH}}\\&=\overrightarrow{\mathrm{OI}}-\left(\overrightarrow{\mathrm{OI}}\cdot\frac{1}{b}\vec{f}\right)\frac{1}{b}\vec{f}\\\overrightarrow{\mathrm{LI}}&=\overrightarrow{\mathrm{OI}}-\overrightarrow{\mathrm{OL}}\\&=\overrightarrow{\mathrm{OI}}-\left(\overrightarrow{\mathrm{OI}}\cdot\frac{1}{a}\vec{g}\right)\frac{1}{a}\vec{g}\end{aligned}$$

ここで，$\frac{1}{b}\overrightarrow{f},\ \frac{1}{a}\overrightarrow{g}$ は単位ベクトル，$\overrightarrow{\mathrm{OI}}=k\cdot\frac{1}{b}\overrightarrow{f}+l\cdot\frac{1}{a}\overrightarrow{g}$ と表せて，

$\left|\overrightarrow{\mathrm{HI}}\right|=\left|\overrightarrow{\mathrm{LI}}\right|$ より，$k=l$（(3)の結果）

$$\therefore\ \overrightarrow{\mathrm{OI}}=k\left(\frac{1}{b}\vec{f}+\frac{1}{a}\vec{g}\right)\ (k>0)\text{ と表せる．}$$

直線 OI と辺 AB の交点を N とおくとき，

$$\mathrm{AN}=c\cdot\frac{b}{b+a}=\frac{bc}{a+b}$$

よって，

$$\mathrm{OI}:\mathrm{IN}=b:\frac{bc}{a+b}=a+b:c$$

$\overrightarrow{\mathrm{ON}}=\dfrac{ab}{a+b}\left(\dfrac{1}{b}\vec{f}+\dfrac{1}{a}\vec{g}\right)$ より，

$$\begin{aligned}\overrightarrow{\mathrm{OI}}&=\frac{a+b}{a+b+c}\overrightarrow{\mathrm{ON}}\\&=\frac{a+b}{a+b+c}\cdot\frac{ab}{a+b}\left(\frac{1}{b}\vec{f}+\frac{1}{a}\vec{g}\right)\\&=\frac{1}{a+b+c}\cdot\left(a\vec{f}+b\vec{g}\right)\end{aligned}$$

（答）