$\boxed{\text{I}}$

(1)

(a) $|z_1|=\sqrt{\left(\dfrac{3}{2}\right)^2+2^2}=\dfrac{\overset{ア}{\boxed{5}}}{\underset{イ}{\boxed{2}}}$ (答)

$z_1=\dfrac{5}{2}\left(\dfrac{3}{5}+\dfrac{4}{5}i\right)=\dfrac{5}{2}(\cos\alpha+i\sin\alpha)$ $\quad\therefore \begin{cases}\cos\alpha=\dfrac{3}{5}\\ \sin\alpha=\dfrac{\overset{ウ}{\boxed{4}}}{\underset{エ}{\boxed{5}}}\end{cases}$ (答)

(b)

$$z_3=\{\cos(-\alpha)+i\sin(-\alpha)\}z_2$$

$$=\left(\frac{3}{5}-\frac{4}{5}i\right)\left(\frac{3}{5}+\frac{9}{5}i\right)=\frac{\overset{オ}{\boxed{9}}}{\underset{カ}{\boxed{5}}}+\frac{\overset{キ}{\boxed{3}}}{\underset{ク}{\boxed{5}}}i$$ (答)

(c)

$$z_4=\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)\left(\frac{3}{5}+\frac{9}{5}i\right)=\frac{\overset{ケコ}{\boxed{-9}}}{\underset{サ}{\boxed{5}}}+\frac{\overset{シ}{\boxed{3}}}{\underset{ス}{\boxed{5}}}i$$ (答)

(d) $\mathrm{P}(w)$ に対して,

$$|w-z_3|:|w-z_4|=1:2$$

$$\Leftrightarrow\quad 2\left|w-\left(\frac{9}{5}+\frac{3}{5}i\right)\right|=\left|w-\left(-\frac{9}{5}+\frac{3}{5}i\right)\right|$$

$$\therefore\ 4\left|w-\left(\frac{9}{5}+\frac{3}{5}i\right)\right|^2=\left|w-\left(-\frac{9}{5}+\frac{3}{5}i\right)\right|^2$$

$$\Leftrightarrow\quad \left|w-\left(3+\frac{3}{5}i\right)\right|^2=\frac{144}{25}\quad\therefore\ \left|w-\left(3+\frac{3}{5}i\right)\right|=\frac{12}{5}$$

$\therefore$ (中心) $\overset{セ}{\boxed{3}}+\dfrac{\overset{ソ}{\boxed{3}}}{\underset{タ}{\boxed{5}}}i$, (半径) $\dfrac{\overset{チツ}{\boxed{12}}}{\underset{テ}{\boxed{5}}}$ の円 (答)

(2)

(a)

$$\int_0^a(x^3-a^2x)dx=\left[\frac{x^4}{4}-\frac{a^2}{2}x^2\right]_0^a=-\frac{a^4}{4}\ \ (=pa^q)$$

$\therefore\ p=\dfrac{\overset{アイ}{\boxed{-1}}}{\underset{ウ}{\boxed{4}}}$, $q=\overset{エ}{\boxed{4}}$ (答)

$$\left|\int_0^a x(x^2-a^2)dx\right|=\left|-\frac{a^4}{4}\right|=\frac{a^4}{4}=4$$

$a\geqq 0$ より,

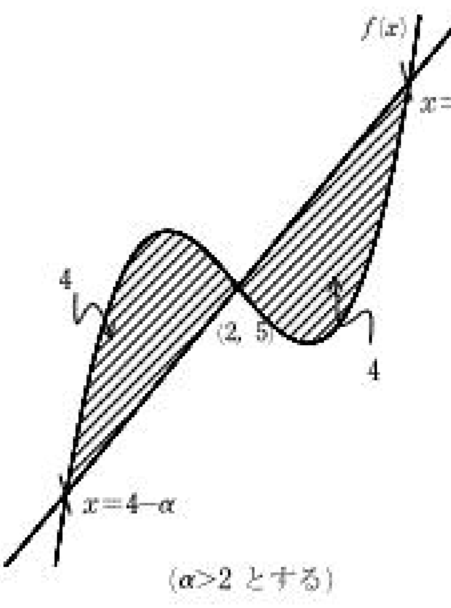
$a=\overset{オ}{\boxed{2}}$ (答)

(b)

$f(x)=x^3-6x^2+10x+1$
$f'(x)=3x^2-12x+10$
$f''(x)=6x-12$

$x=2$ の前後で $f''(x)$ の符号変化

$\therefore$ 変曲点 $\left(\overset{カ}{\boxed{2}},\overset{キ}{\boxed{5}}\right)$ (答)



($\alpha>2$ とする)

$$\left|\int_2^{\alpha}(x-2)(x-4+\alpha)(x-\alpha)dx\right|$$

$t=x-2$
$\therefore\ dt=dx$

x	$2 \to \alpha$
t	$0 \to \alpha-2$

$$=\left|\int_0^{\alpha-2}t(t-2+\alpha)(t+2-\alpha)dt\right|$$

$$=\left|\int_0^{\alpha-2}t\{t^2-(\alpha-2)^2\}dt\right|=4$$

(a)より, $\alpha-2=2\ \therefore\ \alpha=\overset{ク}{\boxed{4}}$ (答)

2 点 $(2, 5)$, $(4, 9)$ を通る直線とわかる.

$y=\dfrac{9-5}{4-2}(x-2)+5\quad\therefore\ y=\overset{ケ}{\boxed{2}}x+\overset{コ}{\boxed{1}}$ (答)

(3)

(a)

$$\overline{x^2}-(\overline{x})^2$$

$$=\frac{1}{2n+1}\{(-n)^2+(-n+1)^2+\cdots\cdots+(n-1)^2+n^2\}-\frac{1}{(2n+1)^2}\underbrace{\{(-n)+(-n+1)+\cdots\cdots+(n-1)+n\}^2}_{0}$$

$$=\frac{1}{2n+1}\cdot\frac{2}{6}n(n+1)(2n+1)$$

$=\dfrac{n}{3}(n+1)=\dfrac{56}{3}\qquad\therefore\ n=\overset{ア}{\boxed{7}}$ (答)

(b)

$$\begin{cases}a_1<a_2<a_3\\ \dfrac{1}{3}(a_1+a_2+a_3)=1\\ \dfrac{1}{3}(a_1{}^2+a_2{}^2+a_3{}^2)-1^2=\dfrac{26}{3}\end{cases}\Leftrightarrow\begin{cases}a_1<a_2<a_3 & \cdots\cdots①\\ a_1+a_2+a_3=3 & \cdots\cdots②\\ a_1{}^2+a_2{}^2+a_3{}^2=29 & \cdots\cdots③\end{cases}$$

15 個のデータは $-7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7$ ……④

③より,

$(a_1, a_2, a_3)=(\pm2, \pm3, \pm4)$ または $(0, \pm2, \pm5)$ (複号任意)

①, ②より,

$(a_1, a_2, a_3)=(-3, 2, 4)$ または $(-2, 0, 5)$

$\therefore\ a_3=\overset{イ}{\boxed{4}}$ または $\overset{ウ}{\boxed{5}}$ (答)

(c) ④のデータから, $(a_1, a_2, a_3)=(-2, 0, 5)$ を取り除くと,

$-7, -6, -5, -4, -3, -1, 1, 2, 3, 4, 6, 7$ の 12 個

平均値 $\overline{x}$ と分散 s^2 は,

$$\overline{x}=\frac{1}{12}\{(-7)+(-6)+(-5)+(-4)+(-3)+(-1)+1+2+3+4+6+7\}=\frac{\overset{エオ}{\boxed{-1}}}{\underset{カ}{\boxed{4}}}$$ (答)

$$s^2=\overline{x^2}-(\overline{x})^2=\frac{1}{12}\{49+36+25+16+9+1+1+4+9+16+36+49\}-\left(-\frac{1}{4}\right)^2$$

$$=\frac{251}{12}-\frac{1}{16}=\frac{\overset{キクケコ}{\boxed{1001}}}{\underset{サシ}{\boxed{48}}}$$ (答)

(4)

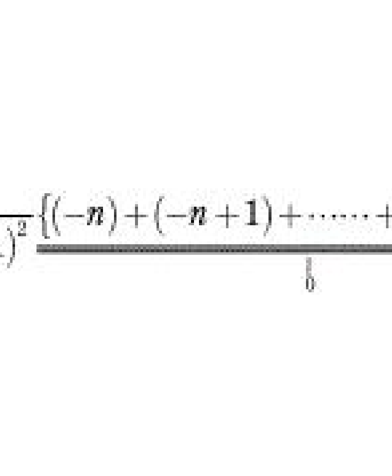
(a)

$$I(x, y)=2x^2+3x+y^2-2y+1$$

$$=2\left(x+\frac{3}{4}\right)^2+(y-1)^2-\frac{9}{8}$$

(最大値) $=I(2, -2)=\overset{アイ}{\boxed{23}}$ (答)

(最小値) $=I\left(-\dfrac{3}{4}, 1\right)=\dfrac{\overset{ウエ}{\boxed{-9}}}{\underset{オ}{\boxed{8}}}$ (答)



(b)

$$(\text{与式})=x^2-4xy+4y^2+2x+y+1$$

$$=(x-2y)^2+2x+y+1$$

$$=X^2+Y+1$$

$\begin{cases}X=x-2y\\ Y=2x+y\end{cases}$ とおくと $\begin{cases}x=\dfrac{X+2Y}{5}\\ y=\dfrac{-2X+Y}{5}\end{cases}$

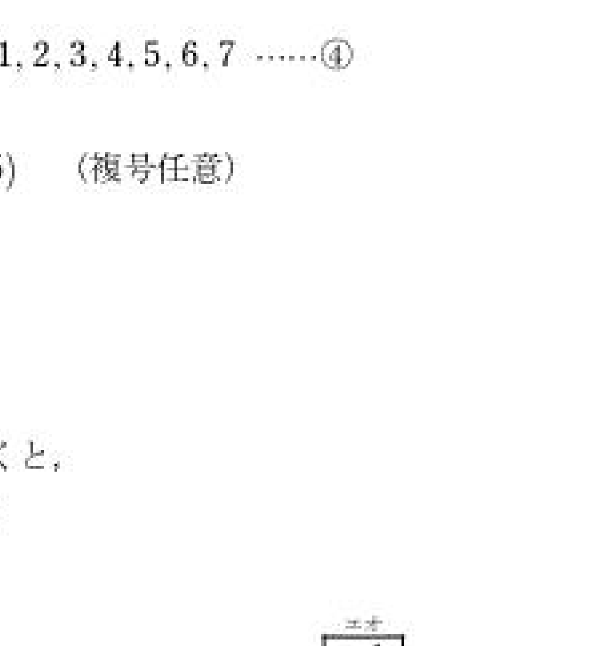
($(x, y) \leftrightarrow (X, Y)$ は 1 対 1 に対応)

ここで,

①を XY 平面に図示.

ここで, $X^2+Y+1=k$ とおく (線形計画法)

$(Y=-X^2+k-1)$



(i) (最大値) $(X, Y)=(-6, -2)$

または $(6, 2)$

$(X, Y)=(-6, -2)\cdots\cdots k=35$

$(X, Y)=(6, 2)\quad\cdots\cdots k=39$

$\therefore$ (最大値) $=\overset{カキ}{\boxed{39}}$ (答)

(ii) (最小値) $Y=-X^2+k-1$ と $Y=-\dfrac{1}{2}X-5$ が接するとき

$$-X^2+k-1=-\frac{1}{2}X-5\ \Leftrightarrow\ 2X^2-X-2k-8=0$$

$$D=1-8(-2k-8)=0$$

$\therefore$ (最小値) $=k=\dfrac{\overset{クケコ}{\boxed{-65}}}{\underset{サシ}{\boxed{16}}}$ $(X, Y)=\left(\dfrac{1}{4}, -\dfrac{41}{8}\right)$ (答)

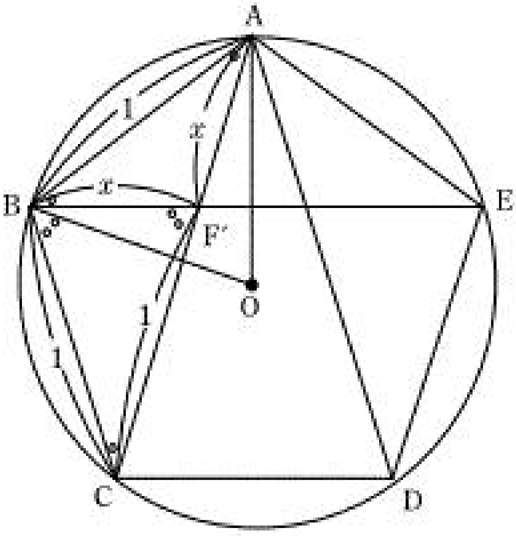
$\boxed{\text{II}}$

(a)　$\angle BAC = 36^\circ = \varphi$ とおく．右図の通り交点 F′ を定めると，
$\triangle F'AB \backsim \triangle BCA$ より，$F'B = F'A = x$ に対して，

$$1 : x = x+1 : 1$$

$$\Leftrightarrow \quad x^2 + x - 1 = 0 \quad \therefore \quad x = \frac{-1+\sqrt{5}}{2}$$

$$\therefore \quad AC = \frac{-1+\sqrt{5}}{2} + 1 = \frac{\overset{\text{ア}}{\boxed{1}} + \sqrt{\overset{\text{イ}}{\boxed{5}}}}{\underset{\text{ウ}}{\boxed{2}}} \qquad \text{(答)}$$



$$\text{次に,} \cos\varphi = \frac{\frac{1}{2}}{x} = \frac{1}{\sqrt{5}-1} = \frac{\sqrt{5}+1}{4}$$

$$\therefore \quad \sin^2\varphi = 1 - \cos^2\varphi = 1 - \left(\frac{\sqrt{5}+1}{4}\right)^2 = \frac{\overset{\text{エ}}{\boxed{5}} - \sqrt{\overset{\text{オ}}{\boxed{5}}}}{\underset{\text{カ}}{\boxed{8}}} \text{(答)}$$

三角形 ABC で正弦定理より，

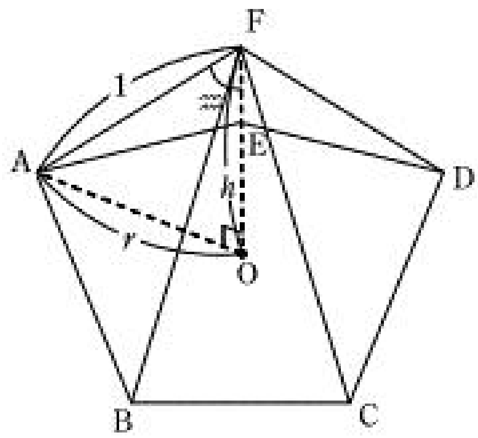
$$\frac{1}{\sin\varphi} = 2r \quad \therefore \quad r^2 = \frac{1}{4}\cdot\frac{1}{\sin^2\varphi} = \frac{\overset{\text{キ}}{\boxed{5}} + \sqrt{\overset{\text{ク}}{\boxed{5}}}}{\underset{\text{ケコ}}{\boxed{10}}} \qquad \text{(答)}$$

また，$S = 5\cdot\frac{1}{2}r^2\sin 2\varphi = 5r^2\sin\varphi\cos\varphi$

$$\therefore \quad S^2 = 25\left(\frac{5+\sqrt{5}}{10}\right)^2 \cdot \frac{5\cdot\sqrt{5}}{8} \cdot \left(\frac{\sqrt{5}+1}{4}\right)^2 = \frac{\overset{\text{サシ}}{\boxed{25}} + \overset{\text{スセ}}{\boxed{10}}\sqrt{\overset{\text{ソ}}{\boxed{5}}}}{\underset{\text{タチ}}{\boxed{16}}} \qquad \text{(答)}$$

(b)　三角形 AOF で，

$$h^2 = 1^2 - r^2 = 1 - \frac{5+\sqrt{5}}{10} = \frac{\overset{\text{ツ}}{\boxed{5}} - \sqrt{\overset{\text{テ}}{\boxed{5}}}}{\underset{\text{トナ}}{\boxed{10}}} \qquad \text{(答)}$$



$$\begin{cases} \sin\theta = \dfrac{r}{1} = r \\ \cos\theta = \dfrac{h}{1} = h \end{cases} \text{より，}$$

$$\sin 2\theta = 2\sin\theta\cos\theta = 2rh = \frac{\overset{\text{ニ}}{\boxed{2}}\sqrt{\overset{\text{ヌ}}{\boxed{5}}}}{\underset{\text{ネ}}{\boxed{5}}} \qquad \text{(答)}$$

(c)　右図より，

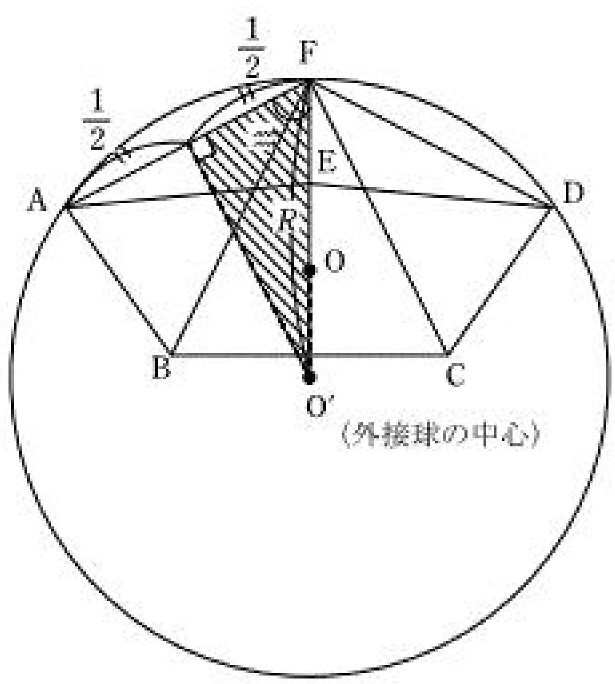
$$\cos\theta = \frac{\frac{1}{2}}{R}$$

$$\Leftrightarrow \quad R = \frac{1}{2\cos\theta}$$

$$= \frac{1}{2}\cdot\frac{1}{\sqrt{\frac{5-\sqrt{5}}{10}}}$$

$$\therefore \quad R^2 = \frac{10}{4\left(5-\sqrt{5}\right)}$$

$$= \frac{\overset{\text{ノ}}{\boxed{5}} + \sqrt{\overset{\text{ハ}}{\boxed{5}}}}{\underset{\text{ヒ}}{\boxed{8}}} \qquad \text{(答)}$$



II

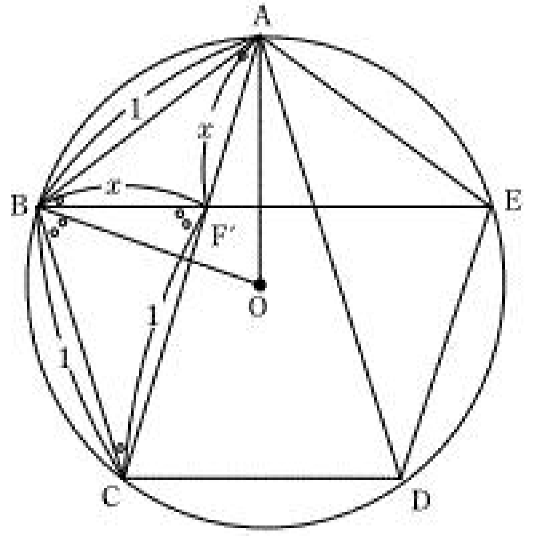
(a)　$\angle BAC = 36^\circ = \varphi$ とおく．右図の通り交点F′を定めると，

$\triangle F'AB \backsim \triangle BCA$ より，$F'B = F'A = x$ に対して，

$$1 : x = x+1 : 1$$

$$\Leftrightarrow \quad x^2 + x - 1 = 0 \quad \therefore \quad x = \frac{-1+\sqrt{5}}{2}$$

$$\therefore \quad AC = \frac{-1+\sqrt{5}}{2} + 1 = \frac{\overset{ア}{\boxed{1}} + \sqrt{\overset{イ}{\boxed{5}}}}{\underset{ウ}{\boxed{2}}} \qquad (答)$$



$$次に, \cos\varphi = \frac{\frac{1}{2}}{x} = \frac{1}{\sqrt{5}-1} = \frac{\sqrt{5}+1}{4}$$

$$\therefore \quad \sin^2\varphi = 1 - \cos^2\varphi = 1 - \left(\frac{\sqrt{5}+1}{4}\right)^2 = \frac{\overset{エ}{\boxed{5}} - \sqrt{\overset{オ}{\boxed{5}}}}{\underset{カ}{\boxed{8}}} \ (答)$$

三角形ABCで正弦定理より，

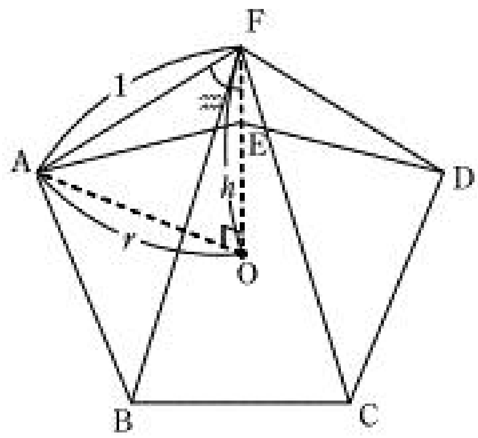
$$\frac{1}{\sin\varphi} = 2r \quad \therefore \quad r^2 = \frac{1}{4}\cdot\frac{1}{\sin^2\varphi} = \frac{\overset{キ}{\boxed{5}} + \sqrt{\overset{ク}{\boxed{5}}}}{\underset{ケコ}{\boxed{10}}} \qquad (答)$$

また，$S = 5\cdot\frac{1}{2}r^2\sin 2\varphi = 5r^2\sin\varphi\cos\varphi$

$$\therefore \quad S^2 = 25\left(\frac{5+\sqrt{5}}{10}\right)^2\cdot\frac{5\cdot\sqrt{5}}{8}\cdot\left(\frac{\sqrt{5}+1}{4}\right)^2 = \frac{\overset{サシ}{\boxed{25}} + \overset{スセ}{\boxed{10}}\sqrt{\overset{ソ}{\boxed{5}}}}{\underset{タチ}{\boxed{16}}} \qquad (答)$$

(b)　三角形AOFで，

$$h^2 = 1^2 - r^2 = 1 - \frac{5+\sqrt{5}}{10} = \frac{\overset{ツ}{\boxed{5}} - \sqrt{\overset{テ}{\boxed{5}}}}{\underset{トナ}{\boxed{10}}} \qquad (答)$$



$$\begin{cases} \sin\theta = \dfrac{r}{1} = r \\ \cos\theta = \dfrac{h}{1} = h \end{cases} より，$$

$$\sin 2\theta = 2\sin\theta\cos\theta = 2rh = \frac{\overset{ニ}{\boxed{2}}\sqrt{\overset{ヌ}{\boxed{5}}}}{\underset{ネ}{\boxed{5}}} \qquad (答)$$

(c)　右図より，

$$\cos\theta = \frac{\frac{1}{2}}{R}$$

$$\Leftrightarrow \quad R = \frac{1}{2\cos\theta}$$

$$= \frac{1}{2}\cdot\frac{1}{\sqrt{\frac{5-\sqrt{5}}{10}}}$$

$$\therefore \quad R^2 = \frac{10}{4\left(5-\sqrt{5}\right)}$$

$$= \frac{\overset{ノ}{\boxed{5}} + \sqrt{\overset{ハ}{\boxed{5}}}}{\underset{ヒ}{\boxed{8}}} \qquad (答)$$