[I]

$$\log_x(1-x^2)>1$$

底 $0<x<1$ より

$$1-x^2<x$$

$$\therefore\ x^2+x-1>0$$

$$\therefore\ x<\frac{-1-\sqrt{5}}{2},\ \frac{-1+\sqrt{5}}{2}<x$$

$0<x<1$ より

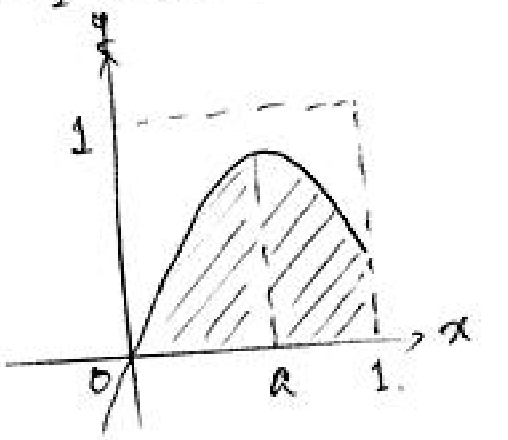
$$\frac{-1+\sqrt{5}}{2}<x<1$$

[II] (1) $y=-\frac{1}{a}x^2+2x$

$=-\frac{1}{a}(x-a)^2+a$

$\therefore$ 頂点 (a,a)

(2) (i) $\frac{1}{2}\leqq a\leqq 1$ のとき.

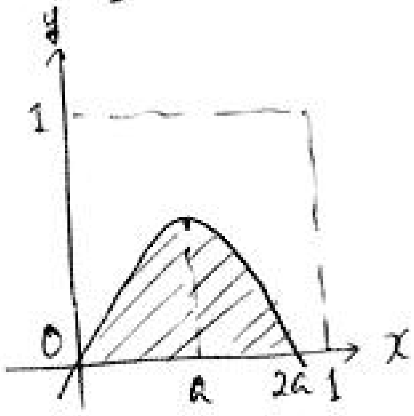


$S=\int_0^1\left(-\frac{1}{a}x^2+2x\right)dx$

$=\left[-\frac{1}{3a}x^3+x^2\right]_0^1=-\frac{1}{3a}+1=\frac{1}{2}$

$\therefore\ a=\frac{2}{3}$ $\left(\frac{1}{2}\leqq a\leqq 1\right.$ を満たす$\left.\right)$

(ii) $0\leqq a\leqq\frac{1}{2}$ のとき.



$S=\int_0^{2a}\left(-\frac{1}{a}x^2+2x\right)dx$

$=\left[-\frac{1}{3a}x^3+x^2\right]_0^{2a}$

$=-\frac{8}{3}a^2+4a^2=\frac{4}{3}a^2=\frac{1}{2}$

$\therefore\ a=\frac{\sqrt{6}}{4}$

これは, $0\leqq a\leqq\frac{1}{2}$ に反するので不適.

よって, 求めるものは $a=\frac{2}{3}$

[III]

① : 45

② : $\sqrt{5}$

③ : $\sqrt{10}$

④ : $\dfrac{7}{2}$

⑤ : $\dfrac{7}{10}\sqrt{2}$