

[I]

$$\cos 2x = -2\sin^2 x + 1 \text{ (I')}$$

$$2 + 3\sin x = -2\sin^2 x + 1$$

$$2\sin^2 x + 3\sin x + 1 = 0$$

$$(\sin x + 1)(2\sin x + 1) = 0$$

$$\therefore \sin x = -1, \frac{1}{2}$$

$$0^\circ \leq x < 360^\circ \text{ (I')}$$

$$\underline{x = 210^\circ, 270^\circ, 330^\circ} \text{ .. (答)}$$

[II]

(1) $y = \frac{1}{2}x^2$, $y = ax + b$ が接するときは

$$\frac{1}{2}x^2 = ax + b$$

$$x^2 - 2ax - 2b = 0 \dots \textcircled{A} \text{ が重解をもつ。}$$

$\textcircled{A}$ の判別式を D_1 とすると

$$D_1/4 = (-a)^2 - 1 \cdot (-2b) = a^2 + 2b = 0$$

$$\therefore b = -\frac{1}{2}a^2 \dots \textcircled{1}$$

また $y = 2x^2 - 12x + 18$, $y = ax + b$ が接するときは

$$2x^2 - 12x + 18 = ax + b$$

$$2x^2 - (a+12)x - b + 18 = 0 \dots \textcircled{B} \text{ が重解をもつ。}$$

$$D_2 = \{-(a+12)\}^2 - 4 \cdot 2 \cdot (-b+18) = 0$$

$$\therefore a^2 + 24a + 8b = 0 \dots \textcircled{2}$$

$$\textcircled{2} \text{ に } \textcircled{1} \text{ を代入して } a^2 + 24a + 8\left(-\frac{1}{2}a^2\right) = 0$$

$$\therefore 3a^2 - 24a = 0$$

$$a(a-8) = 0$$

$$a \neq 0 \text{ より } a = 8$$

$$\therefore \textcircled{1} \text{ に代入して } b = -32$$

$$\therefore \underline{(a, b) = (8, -32)} \dots \text{ (答) }$$

(2) $y = \frac{1}{2}x^2$, $y = 2x^2 - 12x + 18$ の交点の x 座標は

$$\frac{1}{2}x^2 = 2x^2 - 12x + 18$$

$$x^2 - 8x + 12 = 0 \Leftrightarrow (x-2)(x-6) = 0$$

$$\therefore x = 2, 6$$

$$\therefore \text{交点は } (2, 2), (6, 18)$$

$y = \frac{1}{2}x^2$ と $y = 8x - 32$ の接点は

$$\frac{1}{2}x^2 = 8x - 32 \Leftrightarrow (x-8)^2 = 0 \text{ より } (8, 32)$$

$y = 2x^2 - 12x + 18$ と $y = 8x - 32$ の接点は

$$2x^2 - 12x + 18 = 8x - 32 \Leftrightarrow (x-5)^2 = 0 \text{ より } (5, 8)$$

よって求める面積を S とすると

$$S = \int_5^6 \{ (2x^2 - 12x + 18) - (8x - 32) \} dx + \int_6^8 \{ \frac{1}{2}x^2 - (8x - 32) \} dx$$

$$= 2 \int_5^6 (x-5)^2 dx + \frac{1}{2} \int_6^8 (x-8)^2 dx$$

$$= 2 \left[\frac{1}{3}(x-5)^3 \right]_5^6 + \frac{1}{2} \left[\frac{1}{3}(x-8)^3 \right]_6^8$$

$$= 2 \cdot \frac{1}{3} \cdot (6-5)^3 - \frac{1}{2} \cdot \frac{1}{3} \cdot (6-8)^3$$

$$= 2$$

$$\therefore \underline{\text{面積は } 2} \dots \text{ (答) }$$

[III]

$$\textcircled{1} \quad \underline{2p^2 - 2p + 1} \quad \dots \binom{N}{2}$$

$$\textcircled{2} \quad \underline{p^3(1-p)^2} \quad \dots \binom{N}{2}$$

$$\textcircled{3} \quad \underline{p^{n+1}(1-p)^{n-1}} \quad \dots \binom{N}{2}$$

$$\textcircled{4} \quad \underline{6} \quad \dots \binom{N}{2}$$