1

(1) $f(x)=x^2+(2a-1)x+a^2-3a-4$

$$=\left(x+\frac{2a-1}{2}\right)^2-2a-\frac{17}{4}$$

(i) $\frac{1-2a}{2}\leqq 0$ $\left(\frac{1}{2}\leqq a \text{ のとき}\right)$ $\quad f(0)=a^2-3a-4<0$

$$\Leftrightarrow \quad (a+1)(a-4)<0$$

$$\therefore\ -1<a<4 \qquad \therefore\ \underline{\frac{1}{2}\leqq a<4}$$

(ii) $\frac{1-2a}{2}>0$ $\left(\frac{1}{2}>a \text{ のとき}\right)$ $\quad -2a-\frac{17}{4}\leqq 0$

$$\Leftrightarrow \quad -\frac{17}{8}\leqq a \qquad \therefore\ \underline{-\frac{17}{8}\leqq a<\frac{1}{2}}$$

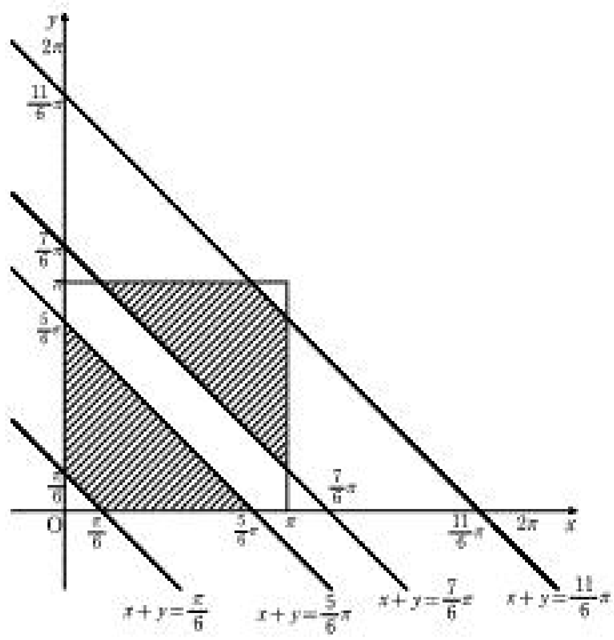
以上 (i), (ii) より $\underline{-\frac{17}{8}\leqq a<4}$ (答)

(2) $\sin(x+y)\leq -\frac{1}{2}, \frac{1}{2}\leq \sin(x+y)$ $\left(\begin{array}{l}\text{ただし}, 0\leqq x\leqq\pi, 0\leqq y\leqq\pi \text{ より}\\ 0\leqq x+y\leqq 2\pi \text{ となる}\end{array}\right)$

より, $\frac{7}{6}\pi\leqq x+y\leqq\frac{11}{6}\pi, \frac{\pi}{6}\leqq x+y\leq\frac{5}{6}\pi$

$\therefore\ -x+\frac{7}{6}\pi\leqq y\leqq -x+\frac{11}{6}\pi, -x+\frac{\pi}{6}\pi\leqq y\leqq -x+\frac{5}{6}\pi$

図示すると右図の斜線部分
ただし, 境界を含む.



(3) 点 $\mathrm{P}_n, \mathrm{P}_{n+1}$ の x 座標は各々 a_n, a_{n+1} より

点 Q_n の x 座標は $\frac{1}{2}(a_n+2)$

点 P_{n+1} の x 座標は $\frac{1}{2}\left\{\frac{1}{2}(a_n+2)+1\right\}=\frac{1}{4}a_n+1$

$\therefore\ \underbrace{a_{n+1}=\frac{1}{4}a_n+1}_{①}$ $\quad(n\geqq 1)$ $\quad$ ただし, $a_1=5$ とする.

ここで, ① $\Leftrightarrow \quad a_{n+1}-\frac{4}{3}=\frac{1}{4}\left(a_n-\frac{4}{3}\right)$

より, 数列 $\left\{a_n-\frac{4}{3}\right\}$ は初項, $a_1-\frac{4}{3}=5-\frac{4}{3}=\frac{11}{3}$, 公比 $\frac{1}{4}$ の等比数列より,

$$a_n-\frac{4}{3}=\frac{11}{3}\left(\frac{1}{4}\right)^{n-1} \Leftrightarrow \underline{a_n=\frac{11}{3}\left(\frac{1}{4}\right)^{n-1}+\frac{4}{3} \ (n\geqq 1)} \quad \text{(答)}$$

$$\therefore\ \underline{\lim_{n\to\infty}a_n=\lim_{n\to\infty}\left\{\frac{11}{3}\left(\frac{1}{4}\right)^{n-1}+\frac{4}{3}\right\}=\frac{4}{3}} \quad \text{(答)}$$

2

(1) 10 個の文字 $\underline{a, a, a}, \underline{b, b}, \underline{c, c}, d, e, f$

$$a, a, a, \underset{a\text{以外の文字}}{\bigcirc} \quad \cdots\cdots \ {}_5\mathrm{C}_1 \times \frac{4!}{3!} = 5 \times 4 = \underline{20 \text{ 通り（答）}}$$

(2) 種類は a, b, c, d, e, f の 6 種類だから

$${}_6\mathrm{P}_4 = 6 \times 5 \times 4 \times 3 = \underline{360 \text{ 通り（答）}}$$

(3) 文字の種類が m 種類 $(m = 2, 3, 4)$ の文字列の数を $n(m)$ と表す．

$$\left.\begin{aligned} n(2) &= 20 + {}_3\mathrm{C}_2 \times \frac{4!}{2!2!} = 20 + 18 = 38 \text{ 通り} \\ n(3) &= {}_3\mathrm{C}_1 \times {}_5\mathrm{C}_2 \times \frac{4!}{2!} = 30 \times 10 \times 12 = 360 \text{ 通り} \\ n(4) &= 360 \text{ 通り} \end{aligned}\right\}$$

全て合わせて $38 + 360 + 360 = \underline{758 \text{ 通り（答）}}$

(1)

$$y=\left(\sqrt{x}-\sqrt{2}\right)^2$$

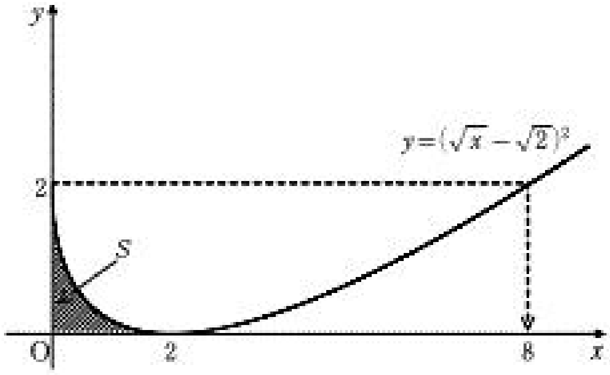
$$=x-2\sqrt{2}\sqrt{x}+2 \quad (x \geqq 0)$$

$$y'=1-2\sqrt{2}\cdot\frac{1}{2}x^{-\frac{1}{2}}$$

$$=1-\frac{\sqrt{2}}{\sqrt{x}}$$

$$=\frac{\sqrt{x}-\sqrt{2}}{\sqrt{x}}$$

x	0	……	2	……	$(+\infty)$
y'		$-$	0	$+$	
y	2	↘	0	↗	$(+\infty)$



グラフを描くと右図の通り．

$$S=\int_0^2\left(\sqrt{x}-\sqrt{2}\right)^2dx$$

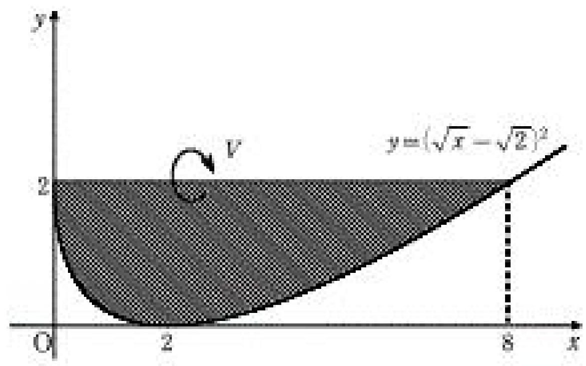
$$=\int_0^2\left(x-2\sqrt{2}\sqrt{x}+2\right)dx$$

$$=\left[\frac{x^2}{2}-2\sqrt{2}\cdot\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+2x\right]_0^2$$

$$=2-\frac{4\sqrt{2}}{3}\cdot2\sqrt{2}+4=\underline{\frac{2}{3}} \quad \text{（答）}$$

(2) $\left(\sqrt{x}-\sqrt{2}\right)^2=2$ とおく．

$\sqrt{x}-\sqrt{2}=\pm\sqrt{2}$ より　$x=0\,,\,8$



$$V=\pi\int_0^8\left\{2-\left(\sqrt{x}-\sqrt{2}\right)^2\right\}^2dx$$

$$=\pi\int_0^8\left(2\sqrt{2}\sqrt{x}-x\right)^2dx$$

$$=\pi\int_0^8\left(8x-4\sqrt{2}x^{\frac{3}{2}}+x^2\right)dx$$

$$=\pi\left[\frac{8}{2}x^2-4\sqrt{2}\cdot\frac{2}{5}x^{\frac{5}{2}}+\frac{1}{3}x^3\right]_0^8$$

$$=\pi\left(\frac{8^3}{2}-\frac{8\sqrt{2}}{5}\cdot\sqrt{8^5}+\frac{8^3}{3}\right)$$

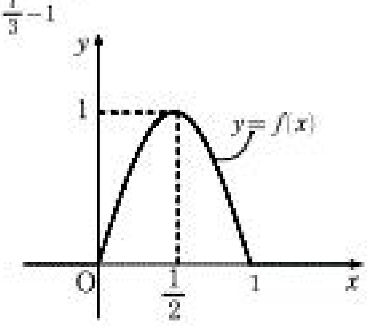
$$=8^3\pi\left(\frac{1}{2}-\frac{4}{5}+\frac{1}{3}\right)$$

$$=\frac{8^3}{30}\pi=\underline{\frac{256}{15}\pi} \quad \text{（答）}$$

$g(t)=\int_0^1 f\left(\frac{t}{3}-x\right)dx$ 　において，$\frac{t}{3}-x=u$ と置換　$\therefore\ -dx=du$

x	$0 \to 1$
u	$\frac{t}{3} \to \frac{t}{3}-1$

$$g(t)=\int_{\frac{t}{3}}^{\frac{t}{3}-1} f(u)\cdot(-1)du=\int_{\frac{t}{3}-1}^{\frac{t}{3}} f(x)dx$$



(i)　$\frac{t}{3}\leqq 0$ ($t\leqq 0$ のとき)　$\underline{g(t)=0}$

(ii)　$0\leqq\frac{t}{3}\leqq 1$　($0\leqq t\leqq 3$ のとき)

$$g(t)=\int_0^{\frac{t}{3}}\sin\pi x dx=\left[-\frac{1}{\pi}\cos\pi x\right]_0^{\frac{t}{3}}=-\frac{1}{\pi}\left(\cos\frac{\pi}{3}t-1\right)$$

$$=\underline{\frac{1}{\pi}\left(1-\cos\frac{\pi}{3}t\right)}$$

(iii)　$0\leqq\frac{t}{3}-1\leqq 1$　($3\leqq t\leqq 6$ のとき)

$$g(t)=\int_{\frac{t}{3}}^{1}\sin\pi x dx=\left[-\frac{1}{\pi}\cos\pi x\right]_{\frac{t}{3}-1}^{1}=-\frac{1}{\pi}\left\{-1-\cos\left(\frac{\pi}{3}t-\pi\right)\right\}$$

$$=\frac{1}{\pi}\left\{1+\cos\left(\pi-\frac{\pi}{3}t\right)\right\}$$

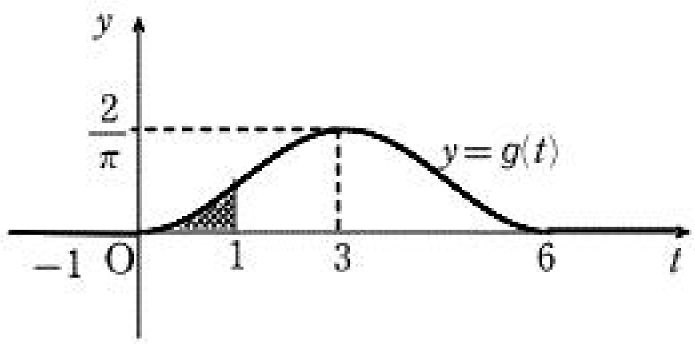
$$=\underline{\frac{1}{\pi}\left(1-\cos\frac{\pi}{3}t\right)}$$

(iv)　$1\leqq\frac{t}{3}-1$　($6\leqq t$ のとき)　$\underline{g(t)=0}$

以上 (i) ～ (iv) をまとめて，

$$\underline{g(t)=\begin{cases}0 & (t\leqq 0\ のとき)\\ \frac{1}{\pi}\left(1-\cos\frac{\pi}{3}t\right) & (0\leqq t\leqq 6\ のとき)\\ 0 & (6\leqq t\ のとき)\end{cases}}\quad (答)$$

$y=g(t)$ のグラフを描くと右図の通り



これより，

$$\int_{-1}^{1}g(t)dt=\int_0^1 g(t)dt=\int_0^1\frac{1}{\pi}\left(1-\cos\frac{\pi}{3}t\right)dt$$

$$=\frac{1}{\pi}\left[t-\frac{3}{\pi}\sin\frac{\pi}{3}t\right]_0^1$$

$$=\frac{1}{\pi}\left(1-\frac{3}{\pi}\cdot\frac{\sqrt{3}}{2}\right)$$

$$=\underline{\frac{2\pi-3\sqrt{3}}{2\pi^2}}\quad (答)$$