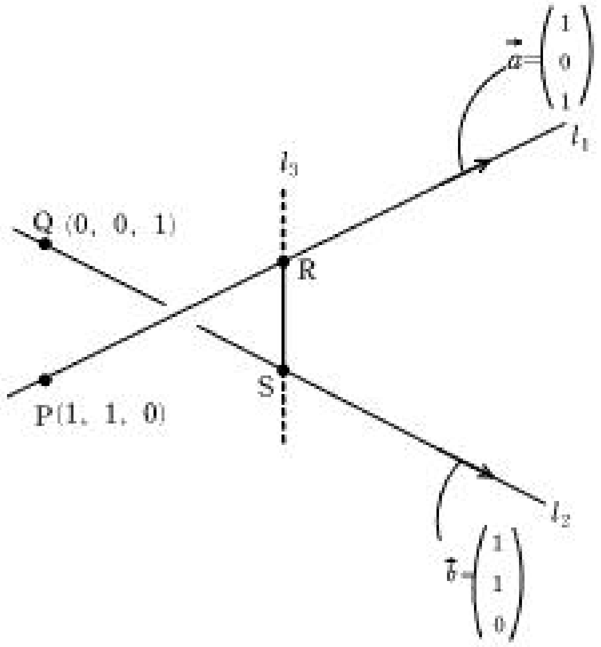
$\boxed{1}$

(1)



$\mathrm{R}(x_1, y_1, z_1), \mathrm{S}(x_2, y_2, z_2)$

$$\begin{pmatrix}x_1\\y_1\\z_1\end{pmatrix}=\begin{pmatrix}1\\1\\0\end{pmatrix}+t_1\begin{pmatrix}1\\0\\1\end{pmatrix}=\begin{pmatrix}1+t_1\\1\\t_1\end{pmatrix}$$

$$\begin{pmatrix}x_2\\y_2\\z_2\end{pmatrix}=\begin{pmatrix}0\\0\\1\end{pmatrix}+t_2\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{pmatrix}t_2\\t_2\\1\end{pmatrix}$$

$$\therefore\ \overrightarrow{\mathrm{RS}}=\begin{pmatrix}t_2-t_1-1\\t_2-1\\1-t_1\end{pmatrix}$$

$l_3 \perp l_1$　かつ　$l_3 \perp l_2$ より，

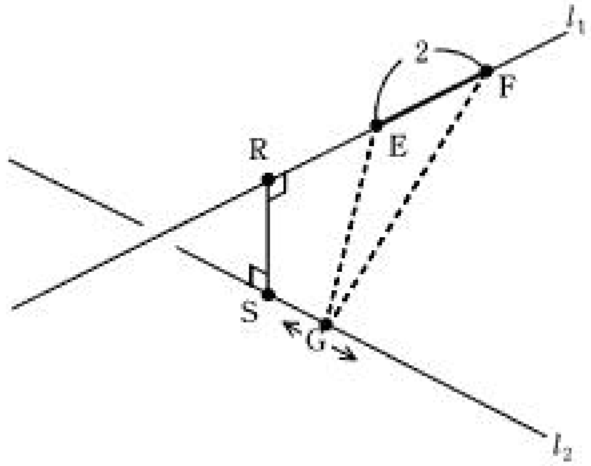
$$\begin{cases}\overrightarrow{\mathrm{RS}}\cdot\vec{a}=0 \iff -2t_1+t_2=0\\ \overrightarrow{\mathrm{RS}}\cdot\vec{b}=0 \iff -t_1+2t_2-2=0\end{cases}$$

$$\therefore\ t_1=\frac{2}{3},\ t_2=\frac{4}{3}$$

よって，

$$\underline{\underline{\mathrm{R}\left(\frac{5}{3}, 1, \frac{2}{3}\right), \mathrm{S}\left(\frac{4}{3}, \frac{4}{3}, 1\right)}}$$

(2)



△EFG が min になるのは EF を底辺とすると高さが min になるとき

そのとき，その高さは

(1)の RS

よって，△EFG の最小値は

$$\frac{1}{2}\cdot 2\cdot\left|\overrightarrow{\mathrm{RS}}\right|=\underline{\underline{\frac{1}{\sqrt{3}}}}$$

$\boxed{2}$

(1)

$$P^{-1}AP=\begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix},\ P^{-1}BP=\begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix}$$

$$P^{-1}AP\cdot P^{-1}BP=\begin{pmatrix} a_1b_1 & a_1b_2 \\ a_2b_3 & a_2b_4 \end{pmatrix}=P^{-1}ABP \qquad \cdots①$$

$$P^{-1}BP\cdot P^{-1}AP=\begin{pmatrix} a_1b_1 & a_2b_2 \\ a_1b_3 & a_2b_4 \end{pmatrix}=P^{-1}BAP \qquad \cdots②$$

$AB=BA$ より，①②の各成分を比較して，

$$\begin{cases} a_1b_2=a_2b_2 \\ a_2b_3=a_1b_3 \end{cases}$$

$a_1 \neq a_2$ より，

$$\underline{\underline{b_2=b_3=0}}$$

(2)

$$A=\begin{pmatrix} 0 & -2 \\ 1 & 3 \end{pmatrix},\ P=\begin{pmatrix} c & 1 \\ -1 & -1 \end{pmatrix}$$

$$P^{-1}=\frac{1}{1-c}\begin{pmatrix} -1 & -1 \\ 1 & c \end{pmatrix}$$

$$P^{-1}AP=\frac{1}{1-c}\begin{pmatrix} -1 & -1 \\ 1 & c \end{pmatrix}\begin{pmatrix} 0 & -2 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} c & 1 \\ -1 & -1 \end{pmatrix}$$

$$=\frac{1}{1-c}\begin{pmatrix} -1 & -1 \\ c & 3c-2 \end{pmatrix}\begin{pmatrix} c & 1 \\ -1 & -1 \end{pmatrix}$$

$$=\frac{1}{1-c}\begin{pmatrix} 1-c & 0 \\ c^2-3c+2 & -2c+2 \end{pmatrix}=\begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}$$

$$c^2-3c+2=(c-1)(c-2)=0 \qquad \therefore\ c=1,\ 2$$

P は逆行列をもつので，$1-c \neq 0$ $\qquad \therefore\ c=2$

$$\underline{\underline{a_1=1,\ a_2=2,\ c=2}}$$

(3)

$$P^{-1}AP=\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

$$P^{-1}BP=\begin{pmatrix} 1 & 1 \\ -1 & -2 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ -1 & -1 \end{pmatrix}$$

$$=\begin{pmatrix} 2 & 2 \\ -1 & -2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ -1 & -1 \end{pmatrix}=\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

$$A^m=P\left(P^{-1}AP\right)^m p^{-1}=\begin{pmatrix} 2 & 1 \\ -1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 2^m \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & -2 \end{pmatrix}$$

$$=\begin{pmatrix} 2 & 2^m \\ -1 & -2^m \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & -2 \end{pmatrix}=\begin{pmatrix} 2-2^m & 2-2^{m+1} \\ -1+2^m & -1+2^{m+1} \end{pmatrix}$$

$$B^m=P\left(P^{-1}BP\right)^m p^{-1}=\begin{pmatrix} 2 & 1 \\ -1 & -1 \end{pmatrix}\begin{pmatrix} 2^m & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & -2 \end{pmatrix}$$

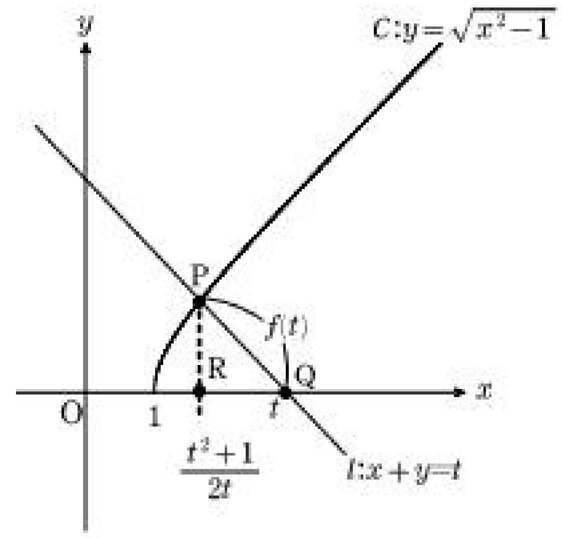
$$=\begin{pmatrix} 2^{m+1} & 1 \\ -2^m & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & -2 \end{pmatrix}=\begin{pmatrix} 2^{m+1}-1 & 2^{m+1}-2 \\ -2^m+1 & -2^m+2 \end{pmatrix}$$

$$\left(A^m+B^m\right)^n=\begin{pmatrix} 2^m+1 & 0 \\ 0 & 2^m+1 \end{pmatrix}^n$$

$$=\underline{\underline{\begin{pmatrix} \left(2^m+1\right)^n & 0 \\ 0 & \left(2^m+1\right)^n \end{pmatrix}}}$$

$\boxed{3}$

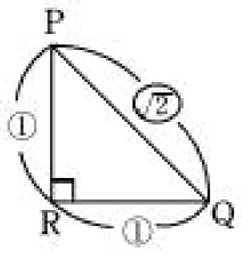
(1)



$$y=t-x=\sqrt{x^2-1}$$

$y>0$ より, $x=\dfrac{t^2+1}{2t}$

$$\mathrm{RQ}=t-\frac{t^2+1}{2t}=\frac{t^2-1}{2t}$$



$$\mathrm{PQ}=f(t)=\sqrt{2}\mathrm{RQ}=\sqrt{2}\cdot\frac{t^2-1}{2t}$$

$$=\frac{1}{\sqrt{2}}\left(t-\frac{1}{t}\right)$$

(2)

$$\lim_{n\to\infty}\sum_{k=1}^{n} f\left(1+\frac{k(t-1)}{n}\right)\frac{t-1}{\sqrt{2}n}$$

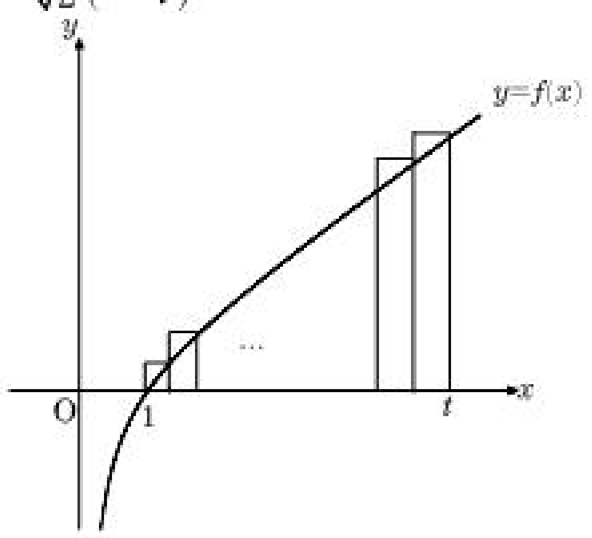
$$=\lim_{n\to\infty}\frac{1}{\sqrt{2}}\cdot\frac{t-1}{n}\sum_{k=1}^{n} f\left(1+\frac{k(t-1)}{n}\right)$$

$$=\frac{1}{\sqrt{2}}\int_1^t f(x)\,dx$$

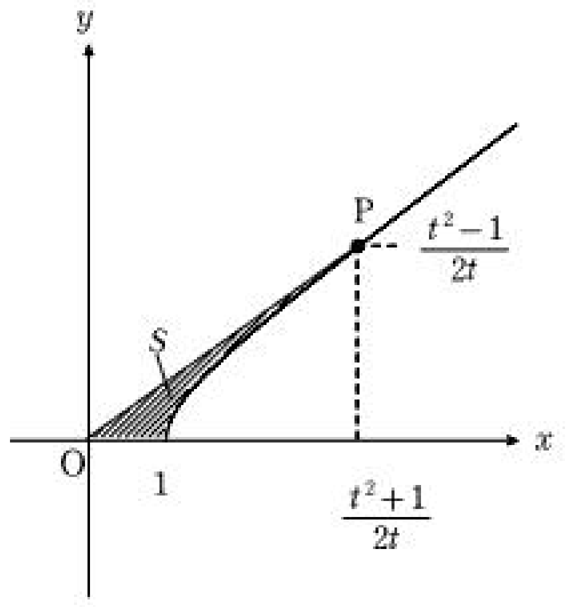
$$=\frac{1}{2}\left[\frac{1}{2}x^2-\log|x|\right]_1^t$$

$$=\frac{1}{4}t^2-\frac{1}{2}\log t-\frac{1}{4}$$

$$f(t)=\frac{1}{\sqrt{2}}\left(t-\frac{1}{t}\right)$$



(3)



$u=x+\sqrt{x^2-1}$ とする.

$$u-x=\sqrt{x^2-1}$$

$$(u-x)^2=x^2-1 \qquad \therefore\ x=\frac{u^2+1}{2u}$$

また、 $\dfrac{du}{dx}=\dfrac{u}{u-x}$ $\qquad \therefore\ dx=\dfrac{u^2-1}{2u^2}du$

x	$1 \to \frac{t^2+1}{2t}$
u	$1 \to t$

$$S=\frac{1}{2}\cdot\frac{t^2+1}{2t}\cdot\frac{t^2-1}{2t}-\int_1^{\frac{t^2+1}{2t}}\sqrt{x^2-1}\,dx$$

$$=\frac{1}{8}\left(t^2-\frac{1}{t^2}\right)-\int_1^t\frac{u^4-2u^2+1}{4u^3}du$$

$$=\frac{1}{8}\left(t^2-\frac{1}{t^2}\right)-\frac{1}{8}\left[u^2-4\log u-u^{-2}\right]_1^t$$

$$=\frac{1}{2}\log t \qquad \therefore\ t=e^{2S}$$

よって,

$$\mathrm{P}\left(\frac{e^{4S}+1}{2e^{2S}},\ \frac{e^{4S}-1}{2e^{2S}}\right)$$