$\boxed{1}$

(1) [1] $n=1$ のとき

$$e^x-\frac{x^1}{1!}=e^x-x=g_1(x),\ g_1{}'(x)=e^x-1$$

$g_1(0)=e^0-0=1$, $x\geqq 0$ のとき $g_1{}'(x)\geqq 0$ より,

$$g(x)\geqq 0\ \ (0\leqq x)$$

$$\therefore\ e^x>\frac{x^1}{1!}$$

[2] $n=k$ のときの成立を仮定する.

$$e^x>\frac{x^k}{k!}\ \cdots\cdots①$$

$g_{k+1}(x)=e^x-\dfrac{x^{k+1}}{(k+1)!}$, $g_{k+1}{}'(x)=e^x-\dfrac{x^k}{k!}$ において,

$g_{k+1}(0)=1$, $g_{k+1}{}'(x)>0$ $(\because ①)$ より,

$$g_{k+1}(x)\geqq 0\ \ (0\leqq x)$$

以上より, $n=k+1$ のとき成立.

[1][2]より, 題意は示せた. //

(2) (1)より, $e^x>\dfrac{x^n}{n!}$ であるから,

$$f(x)=x^{n-1}\cdot e^{-x}=\frac{x^{n-1}}{e^x}<\frac{x^{n-1}}{x^n}\cdot n!=\frac{n!}{x}\to 0\ \ (x\to\infty)$$

$$\therefore\ \lim_{x\to\infty}f(x)=0$$

(3) $f(x)=\dfrac{x^{n-1}}{e^x}$

$$f'(x)=\frac{1}{e^{2x}}\left((n-1)x^{n-2}e^x-x^{n-1}e^x\right)=\frac{1}{e^x}\left((n-1)x^{n-2}-x^{n-1}\right)$$

$$f''(x)=\frac{1}{e^x}\left((n-1)(n-2)x^{n-3}-(n-1)x^{n-2}-(n-1)x^{n-2}+x^{n-1}\right)$$

$$=\frac{x^{n-3}}{e^x}\left(x^2-2(n-1)x+(n-1)(n-2)\right)$$

最大値 $\left(\dfrac{n-1}{e}\right)^{n-1}$ $(x=n-1)$ 変曲点 $x=(n-1)\pm\sqrt{n-1}$

$\boxed{2}$

(1) $\overrightarrow{AO}=-2+4\sqrt{3}\,i$, $\overrightarrow{OB}=3+\sqrt{3}\,i$, $\overrightarrow{BA}=-1-5\sqrt{3}\,i$, $\alpha=\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}=\frac{1}{2}+\frac{\sqrt{3}}{2}i$ とする.

$\overrightarrow{BL}=\alpha\cdot\overrightarrow{BA}=7-3\sqrt{3}\,i$

$\therefore\ \overrightarrow{OL}=\overrightarrow{OB}+\overrightarrow{BL}$

$=10-2\sqrt{3}\,i$

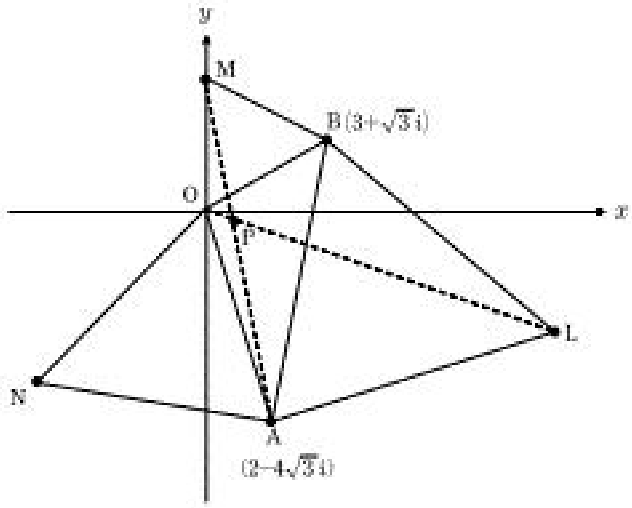
$\overrightarrow{OM}=\alpha\cdot\overrightarrow{OB}=2\sqrt{3}\,i$

$\overrightarrow{AN}=\alpha\cdot\overrightarrow{AO}=-7+\sqrt{3}\,i$

$\therefore\ \overrightarrow{ON}=\overrightarrow{OA}+\overrightarrow{AN}$

$=-5-3\sqrt{3}\,i$

$\therefore\ \underline{\underline{L(10-2\sqrt{3}\,i),\ M(2\sqrt{3}\,i),\ N(-5-3\sqrt{3}\,i)}}$



(2) 座標で考えると, $l_{OL}:y=-\frac{\sqrt{3}}{5}x$, $l_{MA}:y=-3\sqrt{3}x+2\sqrt{3}$

$-\frac{\sqrt{3}}{5}x=-3\sqrt{3}x+2\sqrt{3}\iff x=\frac{5}{7}$

$\therefore\ y=-\frac{\sqrt{3}}{7}$

$\therefore\ \underline{\underline{P\left(\frac{5}{7}-\frac{\sqrt{3}}{7}i\right)}}$

(3) 座標で考えると,

$N(-5,-3\sqrt{3}),\ P\left(\frac{5}{7},-\frac{\sqrt{3}}{7}\right),\ B(3,\sqrt{3})$

$l_{NB}:y=\frac{\sqrt{3}}{2}x-\frac{\sqrt{3}}{2}$ であるが P の座標を代入しても成り立つ.

よって, N, P, B は一直線上. //

$\boxed{3}$

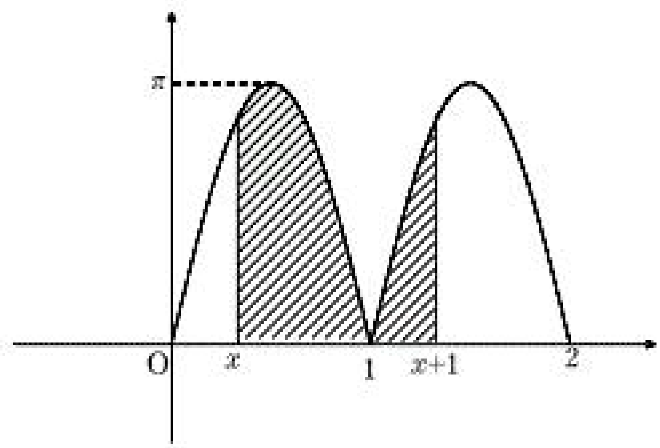
$$\begin{aligned}\int(1-t)e^{1-t}\,dt&=\int e^{1-t}\,dt-\int te^{1-t}\,dt\\&=-e^{1-t}-\left(-te^{1-t}+\int e^{1-t}\,dt\right)\\&=-e^{1-t}+te^{1-t}-(-e^{1-t})\\&=te^{1-t}\end{aligned}$$

(1)　$0\leqq x\leqq 1$

$$\begin{aligned}\int_x^{x+1}|1-t|e^{1-t}\,dt&=\int_x^1(1-t)e^{1-t}\,dt+\int_1^{x+1}(t-1)e^{1-t}\,dt\\&=\left[te^{1-t}\right]_x^1+\left[-te^{1-t}\right]_1^{x+1}\\&=2-xe^{1-x}-(x+1)e^{-x}\\&=\underline{\underline{2-e^{-x}((e+1)x+1)}}\end{aligned}$$

(2)

$$\int_x^{x+1}|\pi\sin(\pi t)|dt=\int_0^1\pi\sin(\pi t)dt=\underline{\underline{2}}$$



(3)　(1)(2)より，

$$f(x)=-e^{-x}((e+1)x+1)$$

$$\begin{aligned}f'(x)&=e^{-x}((e+1)x+1)-e^{-x}(e+1)\\&=e^{-x}((e+1)x-e)\end{aligned}$$

x	0	$\cdots$	$\dfrac{e}{e+1}$	$\cdots$	1
$f'(x)$		$-$	0	$+$	
$f(x)$	-1	↘		↗	$-\dfrac{e+2}{e}$

最大値　-1（$x=0$）

最小値　$-\dfrac{e+1}{e^{\frac{e}{e+1}}}$ $\left(x=\dfrac{e}{e+1}\right)$