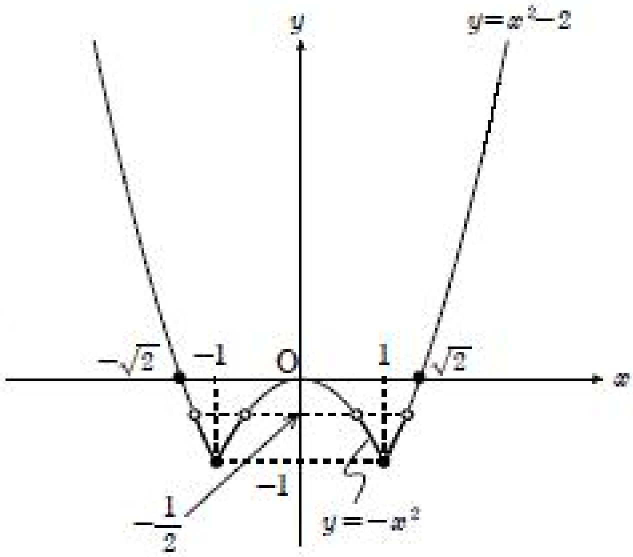
1.

(1)

$$f(x)=\left|x^2-1\right|-1$$
$$=\begin{cases}x^2-2 & (x\leqq-1,\ 1\leqq x)\\ -x^2 & (-1\leqq x\leqq 1)\end{cases}$$

$\therefore\ \underline{\left(f(x)\text{の最小値}\right)=f(\pm1)=-1\quad\left(x=\pm1\text{のとき}\right)}$ (答)



$y=f(x)$ と x 軸との共有点の座標は

$\underline{\left(-\sqrt{2},0\right),(0,0),\left(\sqrt{2},0\right)}$ (答)

(2) $\left|x^2-1\right|<\dfrac{1}{2}\iff\left|x^2-1\right|-1<-\dfrac{1}{2}\iff\underbrace{f(x)<-\dfrac{1}{2}}_{\text{(右上図を参照)}}$

ここで, $-x^2=-\dfrac{1}{2}$, $x^2-2=-\dfrac{1}{2}$ とおくと,

$$x^2=\frac{1}{2},\quad x^2=\frac{3}{2}$$
$$\therefore\ x=\pm\frac{\sqrt{2}}{2},\quad x=\pm\frac{\sqrt{6}}{2}$$

よって,

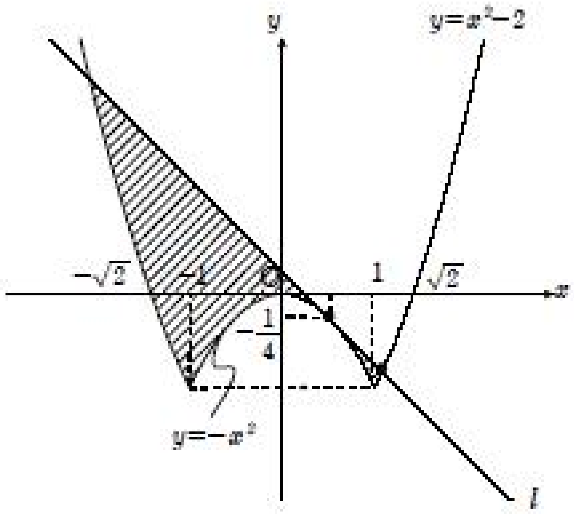
$\underline{-\dfrac{\sqrt{6}}{2}<x<-\dfrac{\sqrt{2}}{2},\ \dfrac{\sqrt{2}}{2}<x<\dfrac{\sqrt{6}}{2}}$ (答)

(3) $y=-x^2$ より, $y'=-2x$

点 $\left(\dfrac{1}{2},-\dfrac{1}{4}\right)$ での接線 l の方程式は,

$$y=-1\cdot\left(x-\frac{1}{2}\right)-\frac{1}{4}$$

$\iff\ \underline{y=-x+\dfrac{1}{4}}$ (答)



(4)

$$x^2-2=-x+\frac{1}{4}$$
$$\iff x^2+x-\frac{9}{4}=0\quad\therefore\ x=\frac{-1\pm\sqrt{10}}{2}$$

求める面積 S は,

$$S=\int_{\frac{-1-\sqrt{10}}{2}}^{\frac{-1+\sqrt{10}}{2}}\left\{-x+\frac{1}{4}-\left(x^2-2\right)\right\}dx-2\int_{-1}^{1}\left\{-x^2-(-1)\right\}dx$$
$$=\frac{1}{6}\left(\sqrt{10}\right)^3-\frac{2}{6}\cdot2^3=\underline{\frac{5\sqrt{10}-8}{3}}\ \text{(答)}$$

2.

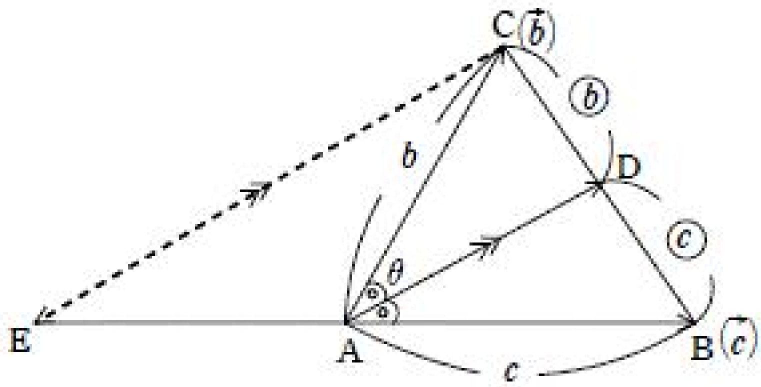
(1) 内角の二等分線の定理より，

$$\mathrm{BD}:\mathrm{DC}=c:b$$

よって，

$$\mathrm{AD}:\mathrm{CE}=c:b+c$$

ここで，$\overrightarrow{\mathrm{AD}}=\dfrac{c\vec{b}+b\vec{c}}{b+c}$ より，



$$\overrightarrow{\mathrm{CE}}=-\frac{c\vec{b}+b\vec{c}}{b+c}\cdot\frac{b+c}{c}$$
$$=\underline{-\vec{b}-\frac{b}{c}\vec{c}}\quad\text{(答)}$$

(2)

$$\cos\theta=\cos 2\left(\frac{\theta}{2}\right)=2\cos^2\frac{\theta}{2}-1=2p^2-1\quad\left(\because\ \cos\frac{\theta}{2}=p\right)$$

$$f^2=\left|\overrightarrow{\mathrm{CE}}\right|^2=\left|\vec{b}+\frac{b}{c}\vec{c}\right|^2$$
$$=b^2+\frac{b^2}{c^2}\cdot c^2+\frac{2b}{c}\vec{b}\cdot\vec{c}$$
$$=2b^2+\frac{2b}{c}b\cdot c\cdot(2p^2-1)$$
$$=4b^2p^2$$

ここで，$0<\theta<\pi$ より，$0<\dfrac{\theta}{2}<\dfrac{\pi}{2}$ つまり，$\cos\dfrac{\theta}{2}=p>0$

$$\therefore\ \underline{f=\left|\overrightarrow{\mathrm{CE}}\right|=2bp}\quad\text{(答)}$$

(3)

$$\overrightarrow{\mathrm{BG}}=\frac{1}{3}\left(\overrightarrow{\mathrm{BE}}+\overrightarrow{\mathrm{BC}}\right)$$
$$=\frac{1}{3}\left(\overrightarrow{\mathrm{BC}}+\overrightarrow{\mathrm{CE}}+\overrightarrow{\mathrm{BC}}\right)$$
$$=\frac{1}{3}\left\{-\vec{b}-\frac{b}{c}\vec{c}+2\left(\vec{b}-\vec{c}\right)\right\}$$
$$=\underline{\frac{1}{3}\left\{\vec{b}-\left(2+\frac{b}{c}\right)\vec{c}\right\}}\quad\text{(答)}$$

(4)

$$\overrightarrow{\mathrm{BG}}\cdot\overrightarrow{\mathrm{AC}}=\frac{1}{3}\left\{\vec{b}-\left(2+\frac{b}{c}\right)\vec{c}\right\}\cdot\vec{b}$$
$$=\frac{1}{3}\left\{b^2-\left(2+\frac{b}{c}\right)\vec{b}\cdot\vec{c}\right\}$$
$$=0$$

これより，

$$\vec{b}\cdot\vec{c}=\frac{b^2}{2+\dfrac{b}{c}}$$

$$\Longleftrightarrow\ bc\cos\theta=\frac{b^2}{2+\dfrac{b}{c}}$$

$$\Longleftrightarrow\ \cos\theta=\underline{\frac{b}{b+2c}}\quad\text{(答)}$$

3.

(1) $x=l$ $\left(l=1, 2, 3, \cdots\cdots, 2k-3,\ l:\text{自然数}\right)$とすると，

①より，　$y+z=2k-1-l$

$\left(y, z\right)$の組の数は，　$2k-2-l$　(個)

よって，

$$\sum_{l=1}^{2k-3}(2k-2-l)=(2k-2)(2k-3)-\frac{(2k-2)(2k-3)}{2}$$
$$=\frac{1}{2}(2k-2)(2k-3)$$
$$=(k-1)(2k-3)$$
$$=\underline{2k^2-5k+3}\ \text{(個)(答)}$$

(2)

$$\sum_{l=1}^{k}(2k-2-l)=(2k-2)k-\frac{k(k+1)}{2}$$
$$=\underline{\frac{k(3k-5)}{2}\ \text{(個)(答)}}$$

(3)　$x=l$ $\left(l=1, 2, 3, \cdots\cdots, k\ \ ,\ l:\text{自然数}\right)$とすると（(1) と同じおき方）

①より，　$y+z=2k-1-l$

$y \leqq k+1,\ z \leqq k+2$に注意して描くと，右図の通り．

$$\sum_{l=1}^{k-4}(l+5)+(k+1)+k+(k-1)+(k-2)$$

↑

$\boxed{\left(k-3-l, k+2\right)\sim\left(k+1, k-2-l\right)\text{の }l+5\text{ (個)}}$

$$(\text{与式})=\frac{(k-4)(k-3)}{2}+5(k-4)+4k-2$$
$$=\underline{\frac{1}{2}(k^2+11k-32)\ \text{(個)(答)}}$$